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CONTENTS

I. Introduction

Inventory costs are substantial. Corporate assets committed to inventories are estimated to exceed $275 billion (based on 1968 figures [3] increased at the same rate as the consumer price index). A firm can utilize a variety of inventory policies to make its product available to the customer, but the costs of these different policies could vary substantially. If inventory carrying charges are assumed to be 25 percent of the value of the inventory then the cost of holding the estimated corporate inventories would exceed $68 billion. A one percent reduction of the present estimated inventory holdings would result in annual savings of over $680 million. This does not include the one time reduction of the capital invested in inventories by over $2.75 billion.

The problem to be studied in this thesis is that of comparing alternative inventory policies for an individual industrial firm. There are many inventory policies that could be used to reach the objectives of the individual firm. The objectives of the firm will vary with the product, the market and the nature of the industry. The procedures presented in this thesis could be used by individual firms to determine if their inventory policies are providing the service required at the lowest possible cost. Their costs may be able to be reduced by changing order quantities (and hence, order frequencies) and/or the levels of safety stock. The interaction of the various service levels and costs will become readily

apparent because alternative cost curves for different inventory policies will be constructed. The firm could use these alternative cost curves in selecting the parameters of the inventory policy that properly match cost and service while meeting the objectives of their firm.

The methodology to be used is to make comparisons of the separate policies utilizing the inventory data of a firm that manufactures and distributes industrial products at 29 locations in the United States. While the majority of the firm's sales are in expendable items, a small portion involves tools and machinery designed to use their supplies and aid in expanding their market. The firm is successful and has experienced an extended period of increased sales. One of the reasons for this growth is the extremely high level of service provided to its customers. The firm's sales are approaching $500 million annually and its total inventories (raw material, in-process and finished goods within and outside the United States) are approximately $135 million. The specific inventory items selected for examination are three of the firm's highest volume items. Stockage levels and the level of customer service at 19 warehouse locations with a total of 50 separate stocking decisions will be reviewed. Since specific data is proprietary, some of the information has been disguised. This will not have any effect on the resulting cost comparisons because comparisons will be given as percentages.

The cost of the current inventory system will be determined and the expected costs of the various alternatives to be examined will be derived based upon the selected sample. Much of this thesis will concentrate on the equal shortage policy first proposed by Gerson and Brown [8]. Their proposal stated that, subject to a constraint on inventory investment, the dollar value of shortages would be minimized by instituting a policy that provided the same number of shortages for all items. The comparison of inventory policies will be based on three specific policies (current method, equal service and equal shortage) but the methodology could be used for other types of inventory policies that a firm may want to consider.

II. Inventory Methods and Literature Review

The purpose of maintaining an inventory is to add time, place and quantity utility to a product. Products usually cannot be sold to the customer if they are not in the right place at the right time and in the right amount. This is the reason that every salesman would like to have a full warehouse in his backyard. This is not normally done because maintaining an inventory costs money. There are basically six types of costs to be considered when deciding on an inventory policy [3,9,10,11,13]. These are:

1) Procurement costs
2) Carrying costs, including opportunity costs
3) Costs of filling orders
4) Stockout costs
5) Inventory control costs
6) Forecasting costs

This research will deal with only procurement, carrying and stockout costs. The primary objective is to obtain the minimum total cost possible while achieving the stated objective (customer service, sales) of the inventory system. The basic inventory decisions that must be made by the firm are 1) how much to order, and then 2) when to order that amount. Minimizing the procurement or order costs could be achieved by ordering the entire anticipated usage at one time. However, it is doubtful that many firms would have the space or the capital to store or buy such a large quantity. Therefore, it is necessary to consider the total cost picture and, thus, to trade off some of the individual costs.

A logical place to start is with the traditional economic order quantity (EOQ). The EOQ balances ordering and carrying costs and has three basic assumptions. The first is that the replenishment leadtime is fixed and known with certainty. The second is that the demand rate is constant and is also known with certainty. The third assumption is that a replenishment order is received before the next order is placed. Ordering costs are a function of the costs associated with placing a single order multiplied by the number of orders placed. For a given annual demand, the smaller the order quantity the larger the number of orders. Carrying costs depend upon the size of the average inventory. The average inventory will be larger the fewer the number of orders. Different order quantities and the average inventory levels are illustrated in Figure II-1. The total annual cost is [11]:

$$TC = \frac{DA}{Q} + \frac{iCQ}{2}$$

where

TC = Total cost
D = Annual demand
A = Cost of placing order
i = Annual inventory carrying rate
C = Individual item value
Q = Order quantity

$\frac{D}{Q}$ is the required number of orders per year to satisfy the demand with $\frac{DA}{Q}$ the cost of placing that number of orders. $\frac{Q}{2}$ is the average inventory and iC is the cost of carrying one unit of inventory for one year. The minimum point on

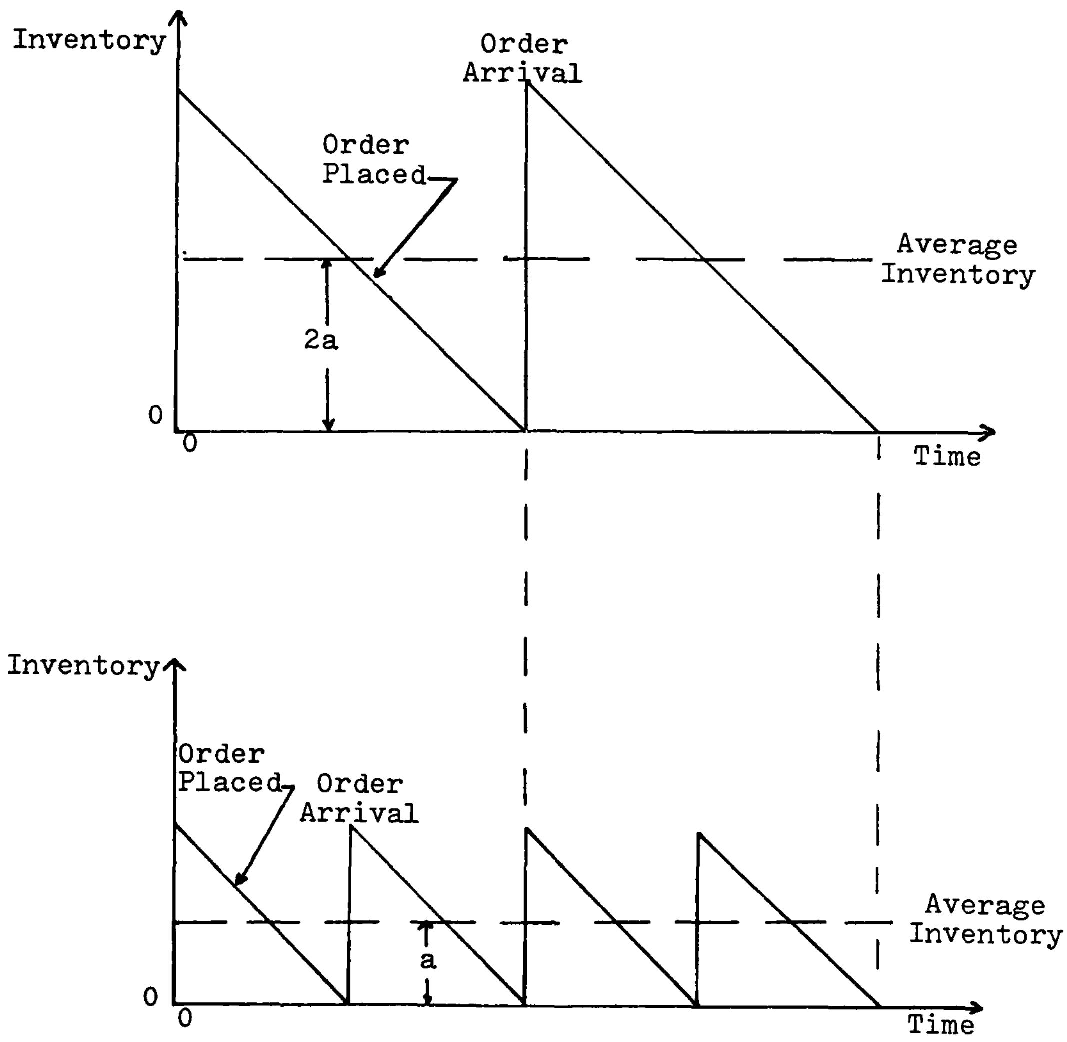

Inventory
Order
Arrival
Order
Placed
Average
Inventory
2a
0
0
Time
Inventory
Order
Placed
Order
Arrival
Average
Inventory
a
0
0
Time

Figure II-1

the total cost curve can be found by taking the first derivative of the total cost equation with respect to Q and setting it equal to zero. This is the point at which a line tangent to the curve has a slope equal to zero and indicates the bottom of the cost curve. It also turns out that the order cost exactly equals the carrying cost at this point. This is shown in Figure II-2.

$$\frac{d(TC)}{d(Q)} = - \frac{DA}{Q} + \frac{iC}{2} = 0$$

Solving, it is found that

$$\text{Optimal Order Quantity } Q^* = \sqrt{\frac{2DA}{iC}}$$

$$\text{Optimal Total Cost } TC^* = \sqrt{2DAiC}$$

One of the difficulties encountered in applying mathematical analysis to inventory management is the estimation of the components of the equations. It is fairly simple to obtain some of the holding cost components such as taxes and insurance on inventories. But it is much more difficult to obtain an accurate figure on other costs. What is the total cost of a stockout? How much does it cost to place an order? These costs will affect inventory decisions but are not the primary concern of this thesis. However, careful observation of Figure II-2 and the equations for the Optimal Order Quantity and Optimal Total Cost show that in the range of the minimum value of the total cost curve its slope is relatively flat. This indicates that a substantial change in either ordering costs or carrying costs would result in only a minor change in the EOQ and also (more

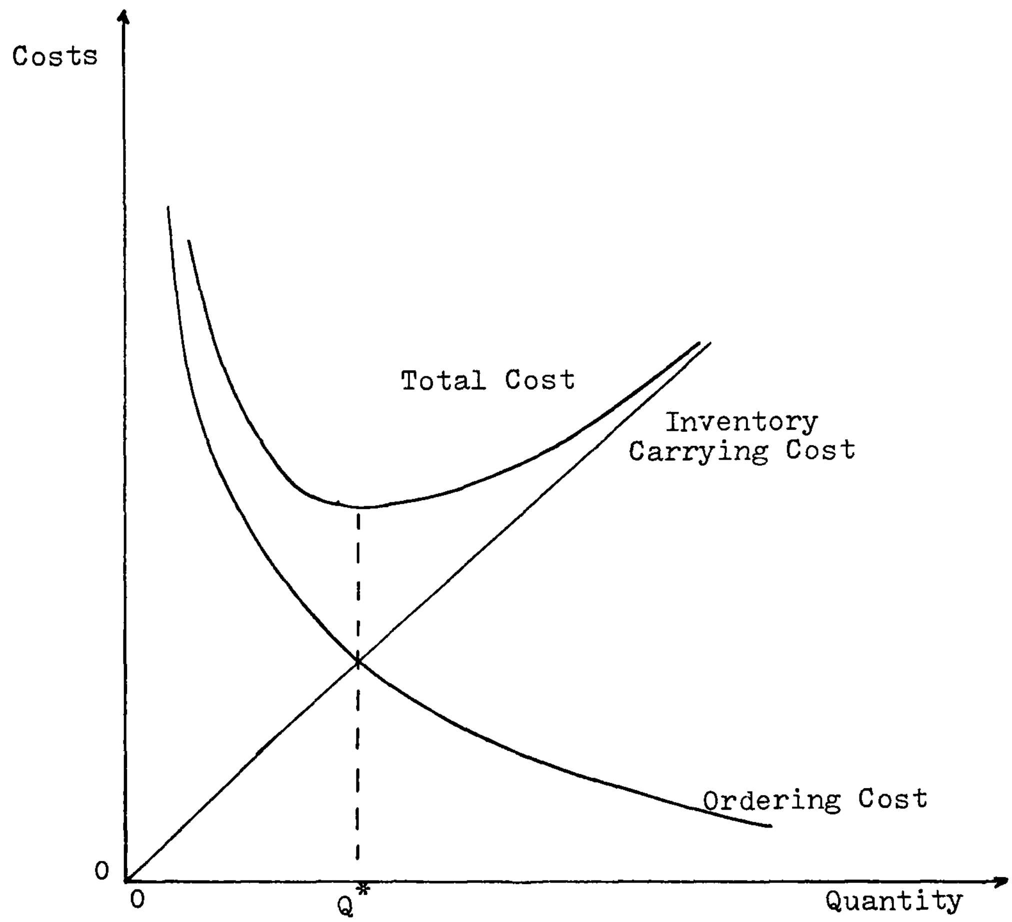

Costs
Total Cost
Inventory
Carrying Cost
Ordering Cost
0
0
Q*
Quantity

Figure II-2

importantly) in the total cost. Consequently, the fact that inventory costs cannot be isolated with exact precision does not dilute the usefulness of the EOQ model.

The time supply method of controlling inventory is probably the simplest of the various inventory methods to understand and will be used to demonstrate some of the basic inventory concepts. The time supply method is based on the average usage of an item. The basic policy is that when the level of inventory falls to the average usage during the required replenishment time, an order is placed. The quantity ordered will be an amount equal to a predetermined time supply, say six months in a deterministic world. When the order quantity arrives the current level of inventory will have dropped to zero, but no stockouts would have occurred because the inventory replenishment arrived at just the right time. This assumes that the usage rate and the replenishment time are known with certainty. However, in reality, either can vary and they are not known with certainty. If the usage rate varied within a normal probability distribution, half of the time the existing level of inventory would be depleted and any demands for an item would result in a stockout condition until the replenishment arrived. This is illustrated in Figure II-3.

This fate, of course, would befall an inventory system regardless of the quantity ordered if the reorder point, the point at which the level of inventory is equal to the

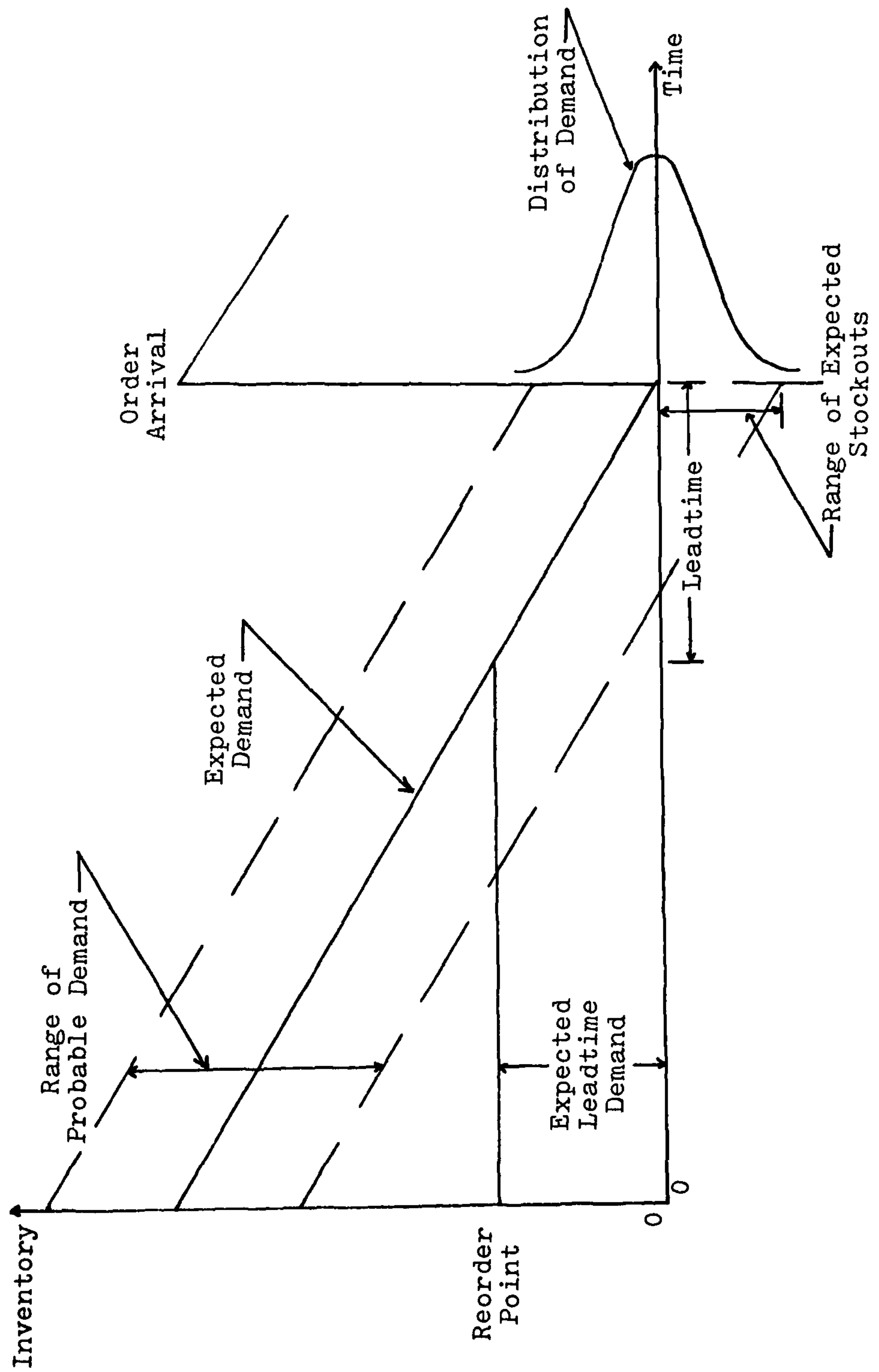
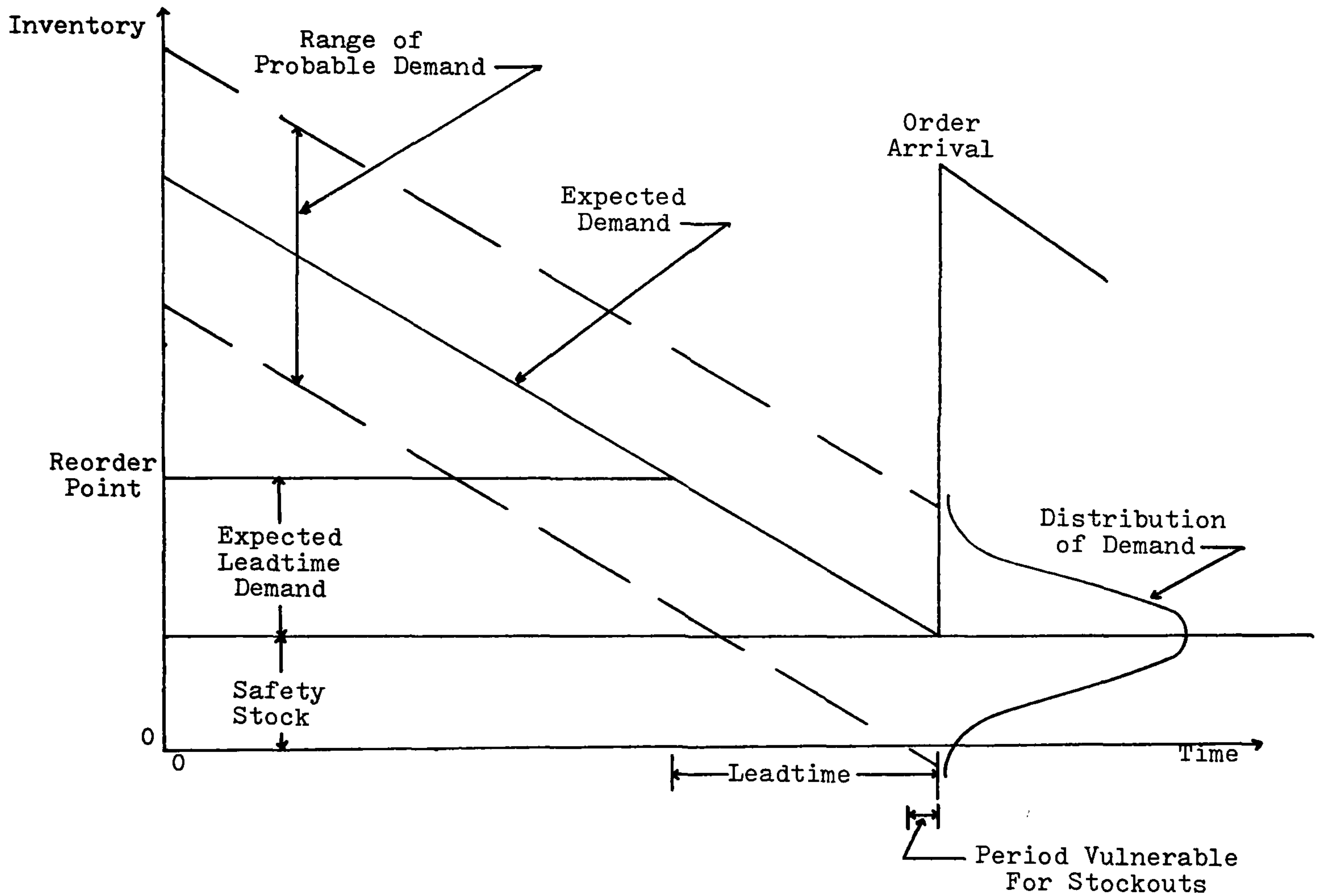

Distribution
of Demand
Time
Order
Arrival
Range of Expected
Stockouts
Leadtime
Expected
Demand
Range of
Probable Demand
Expected
Leadtime
Demand
0
0
Inventory
Reorder
Point

Figure II-3

expected demand during the procurement leadtime, made no allowance for fluctuations of demand or replenishment time. Since it is known that the usage rate and the procurement leadtime will usually vary within limits, a certain amount of protection can be provided. Stockout costs, which consist of such things as lost sales, loss of customer goodwill and expediting replenishments, can be reduced by holding a safety stock (see Figure II-4). But holding a safety stock also costs money because, on average, the safety stock is always held in inventory and is incurring carrying costs. Safety stock will shift the reorder point, the demand line and the average inventory level upward. How far upward depends upon the degree of safety that is to be provided. For example, assuming that demand is either Poisson or normally distributed and the leadtime is a fixed constant, the amount of protection can be stated in terms of the standard deviations of leadtime demand. The assumption of demand being normally distributed is acceptable since most manufacturing situations have approximately normal demands, while a Poisson distribution is characteristic of retail demand [6]. Since stockouts will only be caused by actual demand exceeding expected demand during leadtime it is only necessary to be concerned with one tail of the distribution function. Safety stock is then taken to be:

$$SS = k\sigma_{\bar{X}L}$$



Inventory
Range of
Probable Demand
Order
Arrival
Expected
Demand
Reorder
Point
Expected
Leadtime
Demand
Safety
Stock
Distribution
of Demand
0
0
Time
Leadtime
Period Vulnerable
For Stockouts

Figure II-4

where
k = Safety factor (number of standard deviations)
$\sigma_{\overline{X}L}$ = Standard deviation of leadtime demand

To prevent a stockout in an expected percentage of the order cycles it must be ensured that

$$Pr(XL < \overline{X}L + k\sigma_{\overline{X}L}) \geq 1 - \alpha$$

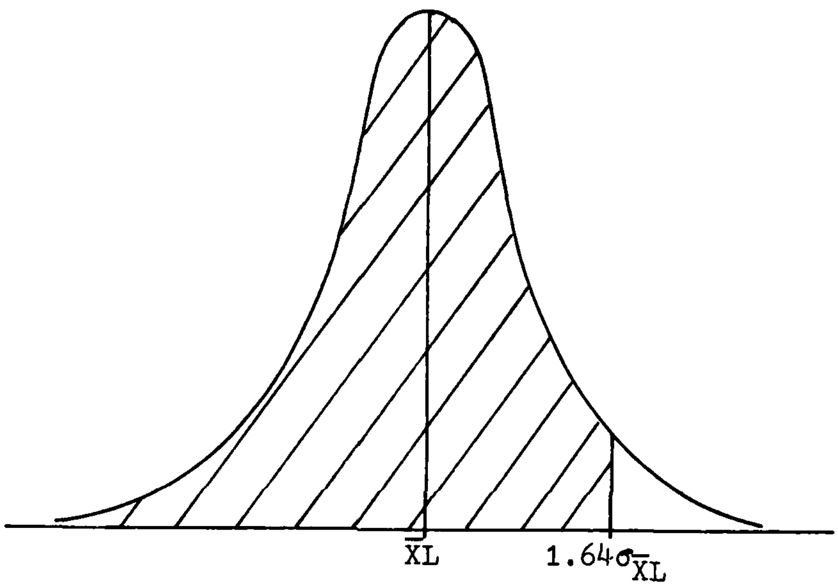
where
$\underline{X}L$ = Actual leadtime demand
$\overline{X}L$ = Expected leadtime demand
α = Probability of a stockout

The safety stock to provide a 5 percent probability of a stockout would be 1.64 standard deviations of the expected leadtime demand (see Figure II-5).

Now that the safety factor is known the reorder point can be determined. The reorder point is that level of inventory at which a replenishment order is initiated. This is the point at which the inventory level is equal to the expected demand during the time until the replenishment is expected to arrive, plus the safety stock. Thus the reorder point is:

$$R = \overline{X}L + SS$$

The reorder point is determined jointly with the safety stock. A set reorder point establishes the level of safety stock and a specific safety factor implies that the reorder point will be at that level that furnishes that amount of safety stock. Safety stock, in units of standard deviations of leadtime demand, is inversely related to the probability of a stockout, as illustrated in Figure II-6. But safety



$\overline{X}L$ = Expected demand during leadtime

$1.64\sigma_{\overline{X}L} = Pr(XL < \overline{X}L + 1.64\sigma_{\overline{X}L}) = .95$

Figure II-5

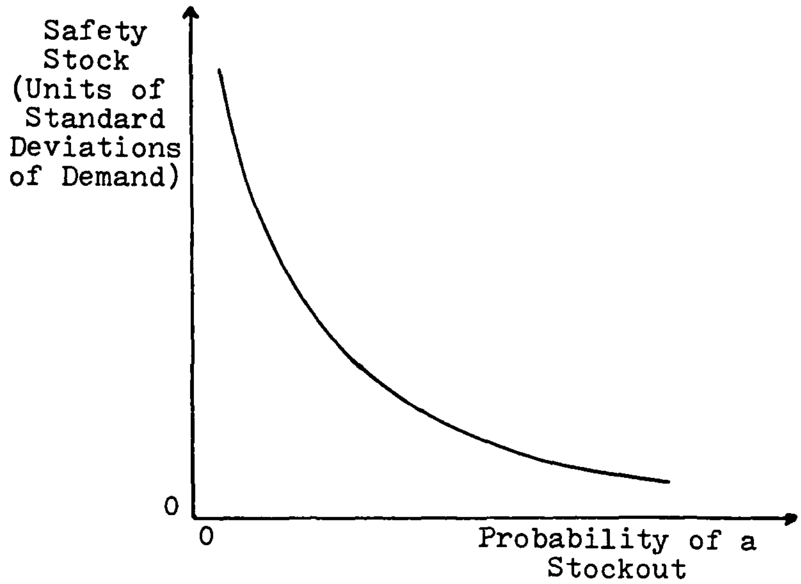


Figure II-6

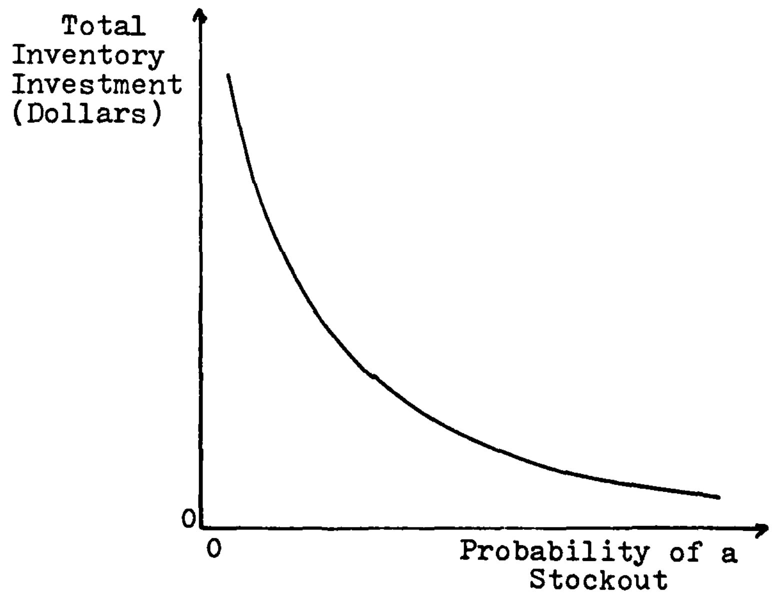


Figure II-7

stock requires an investment in inventories so this leads to the amount of inventory investment also being inversely related to the probability of a stockout, as shown in Figure II-7. If the cost of a stockout can be determined then the proper safety factor would be the point at which the total stockout costs plus the cost of holding safety stock is minimized. Due to the difficulty of assigning a cost to a stockout many firms will establish a policy that sets the probability of satisfying customer demand. This is referred to as the level of customer service.

What is a service level? One definition is that of the probability of not running out of stock. That is:

$$Pr(XL < \bar{X}L + k\sigma_{\overline{XL}}) = 1 - \alpha$$

With no safety stock, as illustrated earlier, the probability of encountering a stockout at the end of the cycle is .50. While no safety stock may indicate that the probability of a stockout condition at the end of any cycle is .50, the level of demand satisfied would be higher, since immediately after receiving a replenishment order the chances of a stockout are usually zero. It is only during the latter part of the inventory cycle that the probability of a stockout occurs. So the protection factor usually is stated for the probability of not encountering a stockout condition only during the leadtime. For example, if the leadtime is one month and the order quantity will provide sufficient stock for an expected six months usage, then

the real probability of a stockout usually occurs during the sixth month. If all of the demand for the first five months is satisfied and the entire demand for the sixth month is backordered, then the actual percentage of demand satisfied will be 83 percent (if the demand in each of the six months is the same). The level of service is commonly computed by defining the probability of having sufficient stock to respond to the demand during the procurement lead-time at the end of the cycle. This type of policy tends to favor the slower moving items because slower moving items generally have a longer cycle time and consequently, fewer cycles per year. With fewer cycles per year these items will experience fewer opportunities of encountering stock-outs, but the percentage of service provided each cycle is the same as for the faster moving items.

This commonly used method of establishing a safety stock to provide for a set probability of stocking out each cycle does not accurately portray the percentage of demand that actually will be satisfied. If the safety stock is established so as to provide a 90 percent opportunity of not running out of stock then, on average, one cycle in every ten will face a stockout. If an item has ten cycles per year then it is probable that a stockout will occur, on the average, of once each year. However, a slower moving item that has only two cycles each year will probably experience a stockout condition, on average,

only once in five years. According to Lewis [14] there are several difficulties with this method of defining service. He indicates that some of the reasons for the unsuitability of this definition are:

1) It does not directly consider ordering, holding or stockout costs.

2) It does not indicate the frequency of stockout conditions.

3) It does not indicate the severity, that is, the amount of units backordered, or the duration of the stockout condition.

4) It does not indicate what proportion of demand will be met from stock.

Brown [4] suggests that a better definition of service would be the proportion of annual demand met from stock. When demand is distributed normally (the same can be applied to any continuous distribution) with a leadtime mean demand of $\overline{X}L$ and a leadtime standard deviation of demand of $\sigma_{\overline{X}L}$ the probability of a stockout, F(k), is the area under the normal distribution curve above $k\sigma_{\overline{X}L}$. That is:

$$F(k) = Pr(XL > \overline{X}L + k\sigma_{\overline{X}L})$$

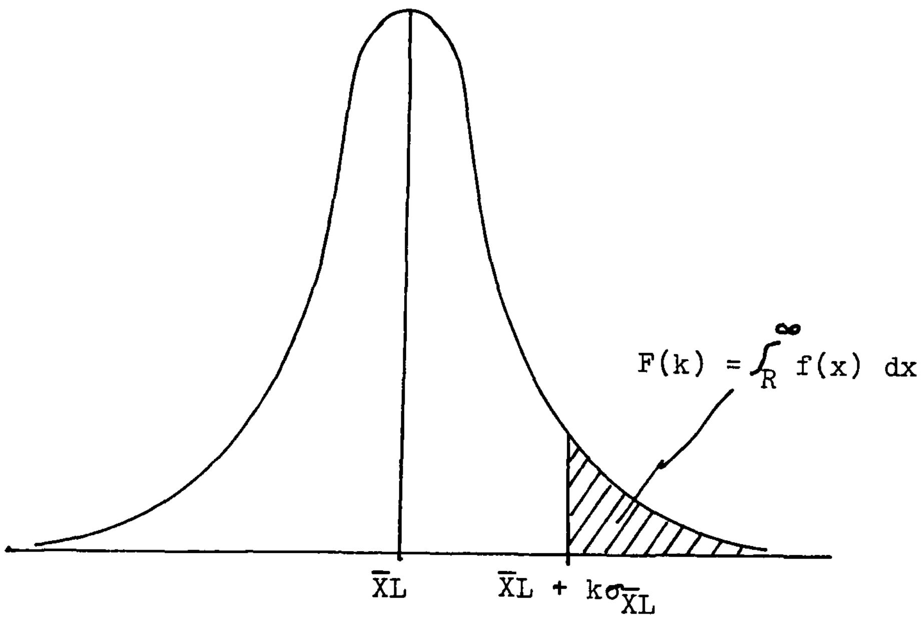
This is illustrated in Figure II-8. This can be found mathematically by taking the integral of the probability density function of leadtime demand

$$F(k) = \int_R^{\infty} f(x)\, dx$$

where

R = Reorder point = $\overline{X}L + SS$

The reorder point being k standard deviations of leadtime



Normal distribution of demand during leadtime, where the integral represents the probability of a stockout and R is the reorder point.

Figure II-8

demand above the expected leadtime demand $\overline{X}L$. The expected number of units backordered is the expected value by which the average or expected demand plus the safety stock ($\overline{X}L$ + $k\sigma_{\overline{X}L}$) will be exceeded. This is known as the "partial expectation" E(k) and is found by taking the integral

$$E(k) = \int_R^\infty (x - R)f(x)dx$$

Both F(k) and E(k) are functions of the safety factor k. Statistical tables for the partial expectation E(k) can be found in several inventory books [4,6,14].

The expected number of items backordered annually can be taken as

$$(\frac{D}{Q})E(k)\sigma_{\overline{X}L}$$

Where $\frac{D}{Q}$ is the number of cycles per year, E(k) is the partial expectation and $\sigma_{\overline{X}L}$ is the standard deviation of leadtime demand. The product, $E(k)\sigma_{\overline{X}L}$, is the expected number of items backordered per cycle. It then follows that the customer service level P, as a percentage of annual demand met from stock, can be found by

$$P = 1 - \frac{E(k)\sigma_{\overline{X}L}}{Q} \qquad (1)$$

Where $\frac{E(k)\sigma_{\overline{X}L}}{Q}$ represents the number of items expected to be backordered per cycle divided by the order quantity (which is the demand for the cycle) to give the percentage of items demanded that are not expected to be filled from stock. The complement of this is the level of demand that is expected to be satisfied from stock. This is a much more

meaningful measure of the service provided by the firm than the previously stated form of measurement of the probability of not stocking out during a particular cycle.

The customer service level equation can be used to illustrate the relationship between the replenishment quantity Q, the reorder point R and the safety factor k. The standard deviation of leadtime demand would remain unchanged. For example, if the replenishment quantity were to be increased resulting in a larger average inventory, but fewer opportunities in which stockouts couldoccur, then the reorder point would be at a lower level of inventory at which the probability of running out of stock in any one cycle would be higher. The two changes would balance each other because, while the expected service level per cycle had decreased, the number of opportunities for a stockout (the number of cycles per year) had been reduced. A safety stock of one standard deviation of leadtime demand would result in the probability of not experiencing a stockout in 84.13 percent of the inventory cycles or stocking out, on the average, every 6.3 cycles. As an example, if the replenishment quantity were 6000 and the standard deviation of leadtime demand was 500, then the expected level of customer service would be:

$$
\begin{aligned}
P &= 1 - \frac{E(k)\sigma_{\overline{X}L}}{Q} \\
&= 1 - \frac{.083(500)}{6000} \\
&= .993
\end{aligned}
$$

Customer demands would be satisfied from stock 99.3 percent of the time. This second method of computation tells much more about the level of service provided by the firm to the customer.

In applying an equal service policy the EOQ would be determined for each item so inventory carrying costs and ordering costs would be properly balanced to yield the lowest total inventory cost. The equation provided earlier would be used.

$$\text{Optimal Order Quantity } Q^* = \sqrt{\frac{2DA}{iC}}$$

The standard deviation of demand during the procurement leadtime must also be estimated. This is simply:

$$\sigma_{\overline{X}L} = \sqrt{\frac{\sum_{j=1}^{n} (XL_j - \overline{X}L)^2}{N - 1}}$$

where

$\sigma_{\overline{X}L}$ = Standard deviation of leadtime demand
XL_j = Actual leadtime demand for item j
$\overline{X}L$ = Expected leadtime demand
N = Number of items

Providing equal service for each item could then be accomplished by establishing the required safety stock for each item. This could be done [7] by rearranging (1) giving:

$$E(k_j) = \frac{Q_j}{\sigma_{\overline{X}Lj}} (1 - P) \tag{4}$$

P is the controlling variable for this policy. Using the same example as previously:

where

$$Q_j = 6000$$
$$\sigma_{\overline{X}Lj} = 500$$
$$P_j = .993$$

This would yield the result:

$$E(k_j) = \frac{6000}{500}(1 - .993)$$
$$= .083$$

When $E(k_j)$ is equal to .083 the safety factor would then be:

$$k_j = 1.00$$

For this item a safety stock of $k_j\sigma_{\overline{X}Lj}$ would equal 500 units and provide an expected customer service level exceeding 99 percent. Setting Q_j and P result in k_j values which provide a total value of backorders. The same process would be followed for all items, but, as the order quantity and the leadtime variance changed, so would the safety factor. However, all items in the inventory would provide the same level of expected service, that is, P would be the same for all items.

While the equal service policy provides the same expected percentage of demand to be filled from stock for all items Gerson and Brown [8] contend that an equal shortage policy may be better. This amounts to solving the following problem for k_j (and sometimes Q_j) without prior reference to a service level P.

$$\text{Minimize} \sum_{j=1}^{n} C_j E(k_j)\sigma_{\overline{X}Lj} D_j/Q_j \qquad (5)$$

$$\text{Subject to} \sum_{j=1}^{n} (k_j\sigma_{\overline{X}Lj}C_j + Q_jC_j/2) \leq I$$

where

$E(k_j)$ = Partial expectation

D_j = Annual demand

I = Inventory investment

The decision variables would be determined by the particular method used to allocate investment in inventory. Their paper presented three sets of decision rules for different (but related) typical situations. All three cases are subject to constraints on inventory investment. The first case has fixed ordering frequencies where every item in stock is ordered with the same frequency. The second case relaxes the restriction on ordering frequencies and allows each item to have its own ordering frequency (and thus solves for Q_j). In the third case ordering costs and shortage costs are considered so as to minimize the total inventory cost.

In the first case, with the ordering frequency fixed, the problem is to minimize the expected dollar value of shortages, that is minimize potential lost revenue, subject to the constraint on safety stock inventory investment (ISS). This can be stated as:

$$\underset{k_j}{\text{Minimize}} \sum_{j=1}^{n} C_j \sigma_{\overline{X}L_j} E(k_j) D_j \,/\, Q_j \tag{6}$$

$$\text{Subject to} \sum_{j=1}^{n} (k_j \sigma_{\overline{X}L_j} C_j) \leq I - \sum_{j=1}^{n} \frac{Q_j C_j}{2} \equiv \text{ISS}$$

The variable k_j is determined by meeting the constraint on

inventory investment. The dollar shortages will be minimized when all items have the same expected number of shortage occurrences per year:

$$\lambda^* = F(k_j^*)\frac{D_j}{Q_j} \qquad (7)$$

where

$F(k_j^*)$ = The probability of a stockout

λ^* = The Langrange multiplier for the constraint

The Langrange multiplier (λ^*) is the effective reduction on lost revenue by increasing inventory investment by \$1. $F(k_j^*)\frac{D_j}{Q_j}$ is the optimal number of shortage occurrences per year. The results are that all items have the same number of shortage occurrences per year and, if all items have the same number of cycles per year, then all safety factors k_j^* are equal.

In the second case the order quantities Q_j^* are to be determined jointly with the safety factors k_j^* to minimize the total value of the shortages subject to a constraint on overall inventory investment. Thus we solve the following problem:

$$\underset{k_j, Q_j}{\text{Minimize}} \sum_{j=1}^{n} C_j \sigma_{\overline{X}L_j} E(k_j) D_j / Q_j \qquad (8)$$

$$\text{Subject to} \sum_{j=1}^{n} (k_j \sigma_{\overline{X}L_j} C_j + Q_j C_j/2) \leq I ,$$

shortages will be minimized if the safety factors satisfy (from 1st order conditions):

$$F^2(k_j^*) = 2\sigma_{\overline{X}L_j} E(k_j^*) \lambda^* / D_j \qquad (9)$$

and the order quantities satisfy:

$$Q_j^* = F(k_j^*)D_j/\lambda^* \tag{10}$$

Combining the two 1st order conditions we have:

$$Q_j^* = \frac{2\sigma_{\overline{X}L_j}E(k_j^*)}{F(k_j^*)} = \sqrt{\frac{2\sigma_{\overline{X}L_j}E(k_j^*)D_j}{\lambda^*}}$$

Now, since $\sigma_{\overline{X}L_j}E(k_j^*)/Q_j^*$ is the percent of demand unsatisfied we have that:

$$\frac{\sigma_{\overline{X}L_j}E(k_j^*)}{Q_j^*} = \frac{\text{Average amount backordered per cycle}}{\text{Average demand per cycle}}$$

$$= \text{Percent of demand unsatisfied}$$

$$= 1 - \text{service level} = 1 - P_j.$$

On the other hand, from the 1st order conditions we see that:

$$\frac{\sigma_{\overline{X}L_j}E(k_j^*)}{Q_j^*} = \frac{F(k_j^*)}{2} \tag{11}$$

Therefore, the relationship between k_j^* and the service level P_j is:

$$F(k_j^*) = 2(1 - P_j) \tag{12}$$

Thus, specifying P_j provides k_j^* which in turn yields Q_j^*. Furthermore, since the probability of a stockout must be less than one, then the level of customer service provided has to be greater than 0.5 and at least half of the value of demand will be satisfied, on the average. For the normal distribution this means that the safety factors are nonnegative.

The last case considers a cost A_j of processing an order for item j and a cost u_j of processing a backorder,

or expediting. The problem is to minimize total costs, including consideration of ordering costs, carrying costs and backorder costs. This can be stated as:

$$\underset{k_j, Q_j}{\text{Minimize}} \sum_{j=1}^{n} \frac{A_j D_j}{Q_j} + \sum_{j=1}^{n} u_j \sigma_{\overline{X}L_j} E(k_j) \frac{D_j}{Q_j} \qquad (13)$$

$$\text{Subject to} \sum_{j=1}^{n} \frac{Q_j C_j}{2} + k_j \sigma_{\overline{X}L_j} C_j \leq I$$

Total costs will be minimized when (from 1st order conditions):

$$\lambda^* = \frac{u_j}{C_j} F(k_j^*) \frac{D_j}{Q_j} \qquad (14)$$

Where $\frac{u_j}{C_j} F(k_j^*) \frac{D_j}{Q_j}$ is the optimal "weighted" number of shortage occurrences per year and λ^* is the reduction in costs caused by increasing the inventory investment by $1. The order quantity is

$$Q_j^* = \sqrt{\frac{2\sigma_{\overline{X}L_j} E(k_j^*) u_j + A_j D_j}{\lambda^* C_j}} \qquad (15)$$

If demand does not vary and $\sigma_{\overline{X}L_j}$ is equal to zero then the order quantity equation reduces to the conventional EOQ. Note that the policy variable λ^* governs the trade-off between invested capital and the ordering expense, or is just the carrying charge. If the ordering costs are zero then the problem reverts to the second case.

A complete study of the three separate cases could not be accomplished as initially planned. Because of the uncertainty of demand being relatively small, and the lack of consideration of leadtime, the second case provides no clear solutions for the available data. The third case considers order costs and backorder costs which were ignored by the

first and second cases. But because the order costs of the firm studied are not known and because of the same difficulties with the relatively small variance of demand and the lack of consideration of leadtime, the third case also cannot be applied to the study data.

With the preceding background the equal service and equal shortage policies will be contrasted with an existing industrial inventory system. These efforts will provide some insights into possible alternative policies, and methods of comparing these policies, for providing the appropriate customer service at the minimum cost possible.

III. Comparison of Inventory Methods

A. Background and Assumptions

Inventory investment is an area of increased concern to top level corporate executives. It is an area that could provide substantial cost savings with even a very small percentage reduction of inventories. This is basically because of the large capital investment committed to inventories. The firm that allowed the use of its data for this study maintains total inventories of approximately $135 million. A one percent reduction of its inventory investment would reduce its capital requirement by approximately $1.35 million and its yearly inventory carrying costs (at the low rate of 18 percent) by approximately $243,000.

The firm manufactures and distributes industrial products with total annual sales approaching $500 million. This thesis will examine only three of the firm's highest volume items and the inventory stocking policies of the distribution warehouses that make the final shipments to its customers. Interaction between the manufacturing plants and the distribution warehouses will not be considered, nor will the total inventory of the firm. The total inventory system (raw material, in-process and finished goods) must be considered by the firm but is beyond the scope of this study.

The items to be studied are stocked at 19 separate warehouse locations. If an item had incomplete demand data

or was manufactured at that location, then the item/location was not used. A total of 50 separate item/location stocking decisions were included in the study. The total annual demand for these item/locations is over 40 million units valued at just over $9 million. In comparing the methods studied a price of $1 was used for each item. The firm desired that actual prices not be used, and since the prices for the items were approximately the same, a $1 price will protect the proprietary information and will not affect the results. All comparisons can be made based on the level of inventory, and the effects on inventory investment can be readily determined by applying the actual price to the results.

Each warehouse location updates a central computer inventory file each day through receipts and issues invoices. A centralized inventory replenishment section reviews every warehouse location's inventory status once each week. The replenishment section determines the stock to be ordered and submits shipping documents to the appropriate manufacturing location for shipment to the distribution warehouse. The leadtime is three weeks and includes one week for the review period, one week for order processing between the replenishment section and the manufacturing plant, and one week in-transit time. This three week leadtime is very stable except for minor fluctuations in the in-transit time for various locations. For the purpose of this study, the leadtime will be assumed to be fixed at three weeks.

The general inventory policy followed by the firm is what is commonly referred to as an R,T policy; that is, at time T (the weekly review), an order quantity sufficient to bring the inventory level up to R (the target) will be submitted. The target at the warehouses is normally a 60 day average demand level. This can be altered, if justified. If, for example, sales personnel have added new accounts in the area served by the warehouse and anticipate increased demand, targets may be increased. But, in time, targets would again return to a 60 days average demand level. Most of the 50 item/location targets were at an average 60 days demand based on the last six month average although some targets had recently been increased well above the current 60 day target. The only stated restriction on ordering was that orders be placed in 1000 unit increments because the items were palletized, 1000 to a pallet. Since the items selected for this study were high volume items, each location encountered an average weekly demand exceeding 1000 units. Consequently, if the stated ordering policy were strictly followed, each location would have an order placed each week for the preceding week's demand, in 1000 unit increments. The current inventory is used in determining the quantity to order. The current inventory is the total of the on-hand plus in-transit plus on-order quantities. With an order being placed each week and a leadtime of three weeks, several shipments are in the "pipeline" at one time.

This is illustrated in Figure III-1.

The firm does not consider the cost of placing an order in any of its inventory order quantity computations. The transaction documents processed by the individual warehouses already require processing, the replenishment section must review every location every week, and shipments from plant to warehouse are almost exclusively by piggy-back permitting smaller shipments. As a result, each plant usually receives a shipping request for each warehouse it serves every week so the incremental costs of an individual order may be fairly low, but they are still present. No set ordering cost will be used for this study.

Since many computations had to be completed for each policy only an example or two is included in the discussion of the alternative inventory methods, but complete tables for each alternative are included as appendixes. This will leave the discussion of the alternative uncluttered by repititious computations and figures. The alternatives to be discussed include:

1) The base, or current, case
2) An equal service policy
3) An equal shortage policy

Before actual comparison of alternatives begins certain initial assumptions will be stated. The first is that the firm follows its stated ordering policy. The replenishment section does have the prerogative of not ordering, for several reasons. However, as of June 30, 1978, the total current

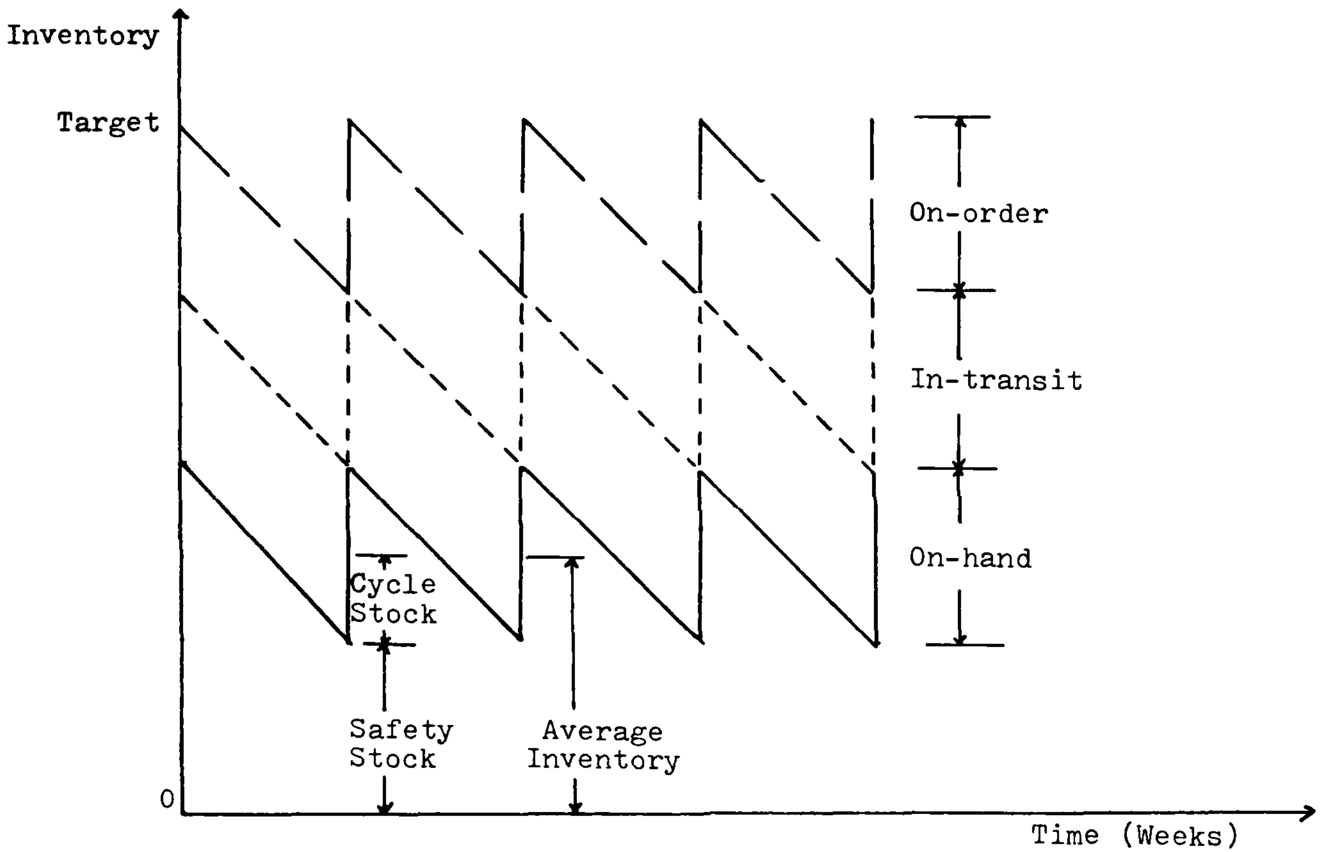

Inventory
Target
0
On-order
In-transit
On-hand
Cycle Stock
Safety Stock
Average Inventory
Time (Weeks)

Figure III-1

inventory was within one percent of the total target. So although deviations are permitted and appear to occur frequently (see Appendix A), it will be assumed that the stated ordering policy of ordering up to the target each week is followed. One liberty which was taken was to use average, or expected, quantities and not to follow the 1000 increment policy. This was done to ease computation. Rather than ordering successive quantities of 5000 and 4000, 4500 would be used. All quantities are rounded to increments of ten.

It is assumed that a linear relationship, over time, exists for the demand data. Demand history was available in monthly increments, leadtime was three weeks and orders were processed weekly. For example, one week's demand is assumed to be one-half of two week's demand. There was no indication that this would not generally be true.

Monthly demand data was tested to determine if any trends or seasonality were present. Demand at some item/locations had increased or decreased but, generally, no trends were indicated, nor, was any seasonality apparent. The Interactive Data Analysis (IDA) program available on Northwestern University's CDC 6600 computer was used to run autocorrelations on the data for 20 item/locations. Since no trends or seasonality were indicated special forecasting methods were not used. The statistics for the six month period July to Decemb.~ 1977 were used in all cases.

It is assumed that the demand data is normally distrib-

uted. A Chi-Square test for goodness of fit was conducted; the results (listed in Appendix B) indicate that the hypothesis that the demand is normally distributed cannot be rejected.

Because the leadtime plus the review period is three weeks, but the order quantities are for one week's demand, it will be assumed, in computing service levels and safety factors, that the three weeks orders are actually one order.

Finally, all comparisons will be based on the inventory level. If an item cost was required, an arbitrary cost of $1.00 was used. This will permit comparisons based on the size of the inventory; actual cost comparisons can easily be obtained by multiplying by the appropriate item cost. Due to rounding some small errors may occur in the totals.

B. Current or Base Case

The total inventory investment will be determined by the size of the average inventory. With the firm's current ordering policy, this is the target minus 2.5 weeks average usage. The 2.5 weeks includes one week for the review period, one week for order processing and one-half of the one week in-transit period. The order arrives at the distribution warehouse two weeks after the review conducted by the inventory replenishment section. However, since the firm includes the one week review period with the leadtime, this study will also include the review period with the delivery leadtime. This can be seen by referring back

to Figure III-1. Stated as an equation:

$$I_a = I_t - 2.5D_w$$

where
I_a = Average inventory
I_t = Inventory target
D_w = Average weekly demand

The average order quantity Q given the current ordering policy, is equal to the average weekly demand. The current average inventory level, using item A1 as an example (see Appendixes A & C) is:

$$\begin{aligned} I_a &= 420{,}000 - 2.5(29{,}980) \\ &= 345{,}050 \end{aligned}$$

Since the average inventory is comprised of the cycle stock and the safety stock, it is a simple task to find the safety stock and consequently, the current level of expected service provided by the firm. Cycle stock (I_c) is one-half of the weekly usage and so is simply:

$$\begin{aligned} I_c &= \frac{D_w}{2} \\ &= \frac{29{,}980}{2} \\ &= 14{,}990 \end{aligned}$$

The average inventory can also be stated as:

$$I_a = I_c + SS$$

This leads to the safety stock being:

$$\begin{aligned} SS &= I_a - I_c \\ &= 345{,}050 - 14{,}990 \\ &= 330{,}060 \end{aligned}$$

The safety factor is equal to:

$$k = \frac{SS}{\sigma_{\overline{X}L}}$$
$$= \frac{330,060}{33,000}$$
$$= 10.0$$

The partial expectation is then:

$$E(k) < .000001$$

Thus, from the relationship between E(k) and customer service: (NOTE: The (3)Q in the denominator of the equation is derived from the assumption considering three separate orders as one order quantity (p.35) and including that as part of (1) from p.20.)

$$P = 1 - \frac{E(k)\sigma_{\overline{X}L}}{(3)Q}$$

We see that:

$$P = 1 - \frac{.000001(33,000)}{89,940}$$
$$= 1 - .00000037$$
$$= .99999963$$

Finally, since

$$E(k) < .000001$$

then

$$P > .99999963$$

The firm provides virtually perfect service for this item. Safety factors for other items vary from .36 to over 53 (see Appendix D). If an item had an expected backorder of less than one unit per cycle, a level of service of 100 percent was used. The total number of expected backorders per cycle is 17,909 and the expected demand per cycle is 2,672,490.

The proportion of stockouts is then:

$$\frac{17,909}{2,672,490} = .0067$$

The level of overall service currently provided by the firm is:

$$1 - .0067 = .9933$$

This level of service is significantly higher than the .95 to .97 probability of not experiencing a stockout that personnel from the firm stated they wished to achieve. This current method will now be contrasted with two alternative methods of allocating inventory; equal service and equal shortage policies.

C. Equal Service Policy

The equal service policy is based upon providing equal levels of service to all items carried in inventory. The ordering policy of the firm studied did not consider the cost of placing an order. To balance inventory carrying costs and ordering costs the EOQ should be used to achieve a total cost that is minimized. Since a figure was not obtained for ordering costs, the current ordering policy of the firm will be used. If an acceptable figure for ordering costs can be determined then the specific order quantities should be re-evaluated and adjusted for minimum total costs. Use of the current ordering policy will allow a very direct comparison of the investment in safety stock. Since the ordering policy will not be changed the order

quantities and the cycle stocks will not be changed. The average inventories, the targets and the safety stocks will all change by the same amount (see Figure III-2). The average inventories, safety stocks, safety factors and required targets for equal service levels of .99, .98, .97, .96, .95, .925, .90 and .85 have been computed for all items (see Appendixes E & F). The computations will be illustrated for only one item at service levels of .99 and .85.

However, before proceeding with the example, three points should be clarified. (1) Because the order quantity is an average of a one week's supply but the leadtime plus review period is three weeks, the order quantity for this study will be considered three times the average order quantity. For this reason the (3)Q in the numerator is considered equivalent to Q as contained in the original equation (1) on p.20. (2) The standard deviation of leadtime demand was calculated by taking the available timeframe (one month) variance of demand and multiplying by .75. This result is the variance for three weeks (one month = four weeks). The square root of this variance is the standard deviation of the leadtime plus review period. (3) The computation of the standard deviation of leadtime demand is acceptable if the covariance is equal to zero. That is, the demand at the various locations is statistically independent from week to week. The nature of the items studied makes this an acceptable assumption.

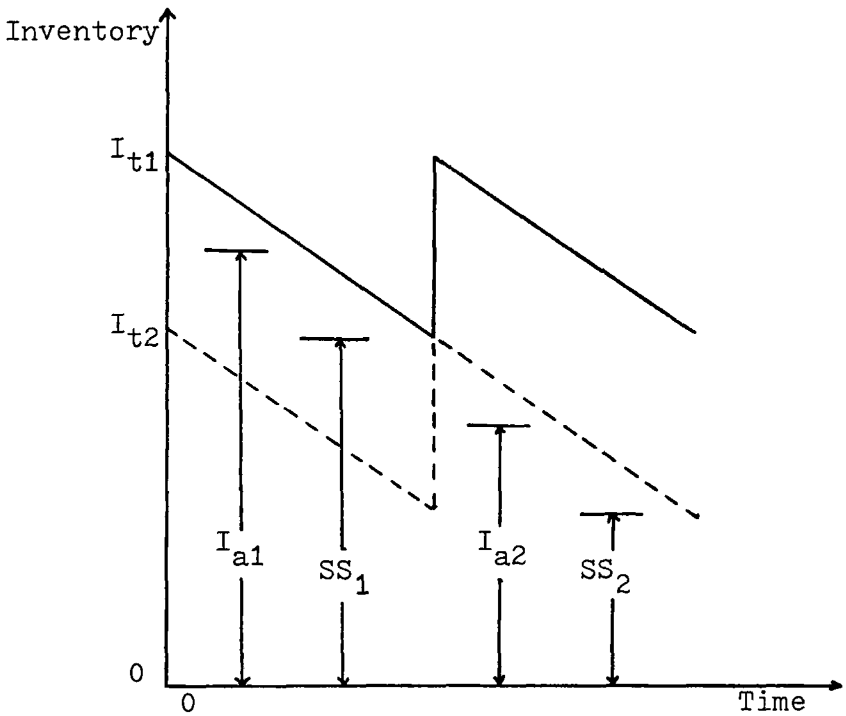


Figure III-2

Once the order quantity has been determined the partial expectation E(k) can be computed (item A1 as an example):

$$E(k) = \frac{(3)Q}{\sigma_{\overline{X}L}}(1 - P)$$

$$= \frac{3(29,980)}{33,000}(1 - .99)$$

$$= 2.73(.01)$$

$$= .0273$$

The safety factor is then:

$$k = 1.53$$

This will lead to the required amount of safety stock:

$$SS = k\sigma_{\overline{X}L}$$

$$= 1.53(33,000)$$

$$= 50,490$$

Which, in turn, leads to the required target level:

$$I_t = SS + \overline{X}L$$

$$= 50,490 + 89,940$$

$$= 140,430$$

The average inventory is then found by:

$$I_a = SS + I_c$$

$$= 50,490 + \frac{29,980}{2}$$

$$= 65,480$$

Using the same item, but providing a much reduced level of expected service (P = .85):

$$E(k) = \frac{(3)Q}{\sigma_{\overline{X}L}}(1 - P)$$

$$= \frac{3(29,980)}{33,000}(1 - .85)$$
$$= 2.73(.15)$$
$$= .4095$$

This would provide the result that:

$$k = < 0$$

This indicates that, although the planned percentage of shortages is 15 percent, better service can be provided with no safety stock at all. With no safety stock, in one-half of the replenishment cycles leadtime demand is less than forecast so there is no shortage. In the other half of the replenishment cycle there will be some shortage, but less than the planned level of shortages. How can this happen? The amount of expected variation of leadtime demand is the reason. For this particular item the standard deviation of leadtime demand is less than one-third of the expected leadtime demand. The purpose of safety stock is to offer protection for the reasonably expected fluctuations of demand. If there were no fluctuations of demand, there would be no need for safety stock. The level of protection offered with no safety stock is still found from the same equation (E(k) for k=0 is .3989):

$$P = 1 - \frac{E(k)\sigma_{\overline{X}L}}{(3)Q}$$
$$= 1 - \frac{.3989(33,000)}{89,940}$$
$$= 1 - .146$$
$$= .854$$

For this item, no safety stock would allow an expected level of service of just over 85 percent. No safety stock would produce the result:

$$I_t = \overline{X}L$$
$$= 89,940$$

and
$$I_a = I_c$$
$$= 14,990$$

Appendixes E & F have the figures for all items. The trade-off between expected service levels and inventory investment for item A1 can be seen in Figure III-3 (item value = $1). Although items were computed to produce equal expected service, at the lower service levels some items received a higher expected service level. This occurred because of instances similar to item A1. Consequently, a number of the items did receive equal service, but a slightly higher service was allowed once the level of safety stock decreased to zero units. For example, 25 of 50 items were able to provide at least 85 percent expected service with no safety stock. The overall service was then expected to be just over 88 percent. As the expected service level increased the number of items able to provide the required expected service with no safety stock decreased. With no safety stock, eight items could provide the expected 90 percent service and three items could provide the expected .925 service. However, all items required safety stock to provide an expected 95 percent service. Thus, for most expected service levels, equal service was provided. The equal service percentages will only be equal, as they were here, if items are ordered with the same frequency.

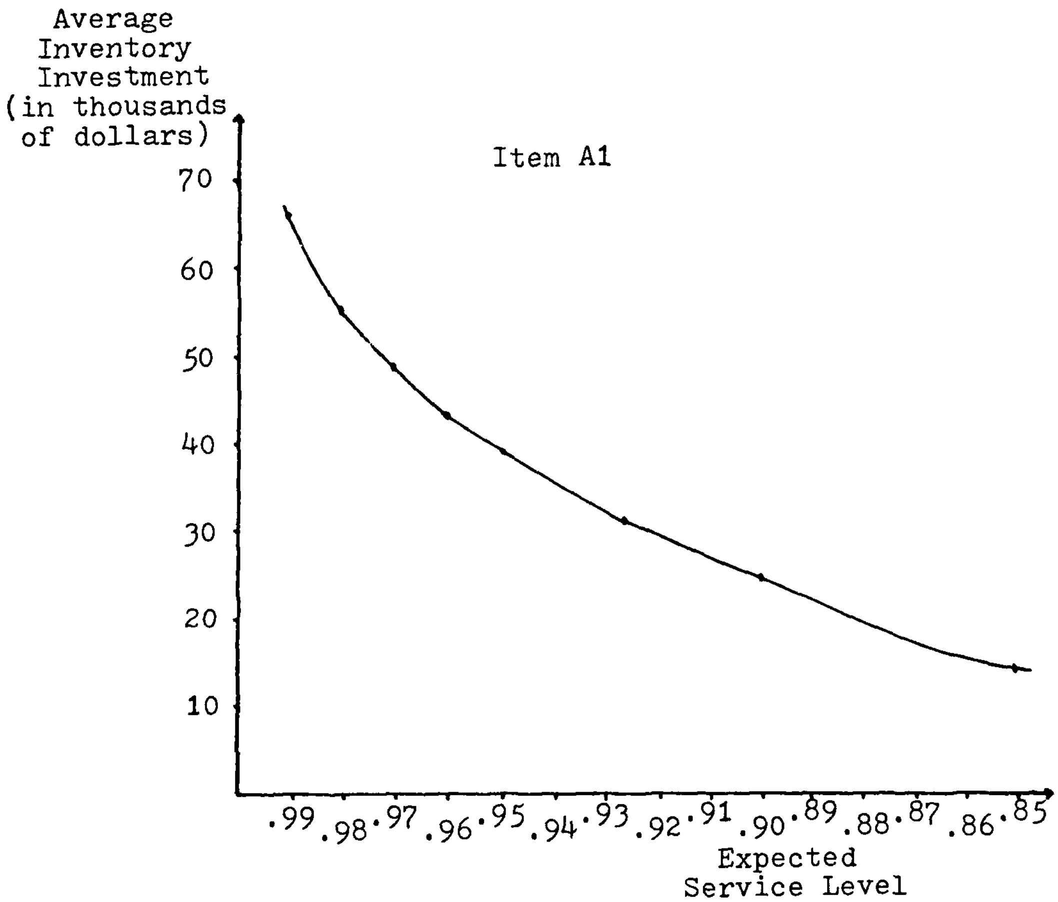

Average
Inventory
Investment
(in thousands
of dollars)
Item A1
70
60
50
40
30
20
10
.99 .98 .97 .96 .95 .94 .93 .92 .91 .90 .89 .88 .87 .86 .85
Expected
Service Level

Figure III-3

D. Equal Shortage Policy

The basic difference between equal service and equal shortage policies is the variable which is held constant. In the equal service policy each item has the expected service level P held constant and the value of the safety factor k will change. The equal shortage policy (the first case) will apply the same safety factor k to each item while the expected individual level of service P will vary.

The first equal shortage situation is where the order quantity is known. The same order quantity as used previously will be continued. For convenience we restate the problem from pp. 24-25:

$$\underset{k_j}{\text{Minimize}} \sum_{j=1}^{n} C_j \sigma_{\overline{X}L_j} E(k_j) D_j / Q_j$$

$$\text{Subject to} \sum_{j=1}^{n} (k_j \sigma_{\overline{X}L_j} C_j) \leq I - \sum_{j=1}^{n} \frac{Q_j C_j}{2} \equiv ISS$$

The total investment in safety stock will be constrained to be equal to the total investment in safety stock of the various expected service levels provided in the equal service cases. With the same total inventory investment, which inventory policy will provide the better expected level of service?

Using the same item, A1, as in the equal service example, the level of safety stock and the target will be computed. Using the same total investment in safety stock that was required in 99 percent equal service, the value of

the safety factor can be computed (see Appendix G for all item computations):

$$\sum_{j=1}^{n} k_j \sigma_{\overline{X}L_j} C_j = ISS$$

However, we assumed C_j to be $1 so this is effectively:

$$\sum_{j=1}^{n} k_j \sigma_{\overline{X}L_j} = ISS$$

But since we will hold k_j constant this results in:

$$k \sum_{j=1}^{n} \sigma_{\overline{X}L_j} = ISS$$

This will lead to the result that:

$$k = \frac{ISS}{\sum_{j=1}^{n} \sigma_{\overline{X}L_j}}$$

$$= \frac{428,460}{266,740}$$

$$= 1.606$$

The level of safety stock for item A1 is then:

$$SS_{A1} = k\sigma_{\overline{X}L_{A1}}$$

$$= 1.606(33,000)$$

$$= 53,000$$

The target can then be found:

$$I_{t_{A1}} = SS_{A1} + \overline{X}L_{A1}$$

$$= 53,000 + 89,940$$

$$= 142,940$$

If $k = 1.606$, then $E(k) = .0229$ and the expected number of

shortages is:

$$E(k)\sigma_{\overline{X}L_{A1}} = .0229(33{,}000)$$
$$= 756$$

This leads to the expected service level:

$$P_{A1} = 1 - \frac{E(k)\sigma_{\overline{X}L_{A1}}}{(3)Q_{A1}}$$
$$= 1 - \frac{756}{89{,}940}$$
$$= .9916$$

The level of expected service is higher than the .99 equal service level. But the level of safety stock is also higher. Will comparison with the .85 equal service level be similar? The equal shortage safety factor is:

$$k = \frac{ISS}{\sum_{j=1}^{n} \sigma_{\overline{X}Lj}}$$
$$= \frac{50{,}480}{266{,}740}$$
$$= .189$$

The level of safety stock for item A1 is then:

$$SS_{A1} = k\sigma_{\overline{X}L_{A1}}$$
$$= .189(33{,}000)$$
$$= 6{,}240$$

The target is then found:

$$I_{t_{A1}} = SS_{A1} + \overline{X}L_{A1}$$
$$= 6{,}240 + 89{,}940$$
$$= 96{,}180$$

If k = .189, then E(k) = .3115 and the expected level of service can be computed:

$$P_{A1} = 1 - \frac{E(k)\sigma_{\overline{X}L_{A1}}}{(3)Q_{A1}}$$

$$= 1 - \frac{.3115(33,000)}{89,940}$$

$$= .8857$$

The level of expected service is higher than was achieved using equal service. With all items having the same reorder frequency the equal shortage policy is in case one, with all safety factors k_j equal. When all ordering frequencies are the same, the dollar value of backorders, for a given inventory investment, is minimized only when all safety factors are equal. If we apply the equal service policy to this situation and have the safety factors not the same, we have a suboptimal solution. That is, the same inventory investment gives a larger value of backorders. Therefore, we would need a greater inventory investment to achieve the same value of backorders. Appendix H contains the equal shortage safety stocks and Appendix I lists the expected backorders for both policies.

The second case of the equal shortage policy cannot be applied to the study data. It will be recalled that:

$$Q_j^* = \frac{2\sigma_{\overline{X}L_j}E(k_j^*)}{F(k_j^*)} = \frac{2\sigma_{\overline{X}L_j}E(k_j^*)D_j}{\lambda^*}$$

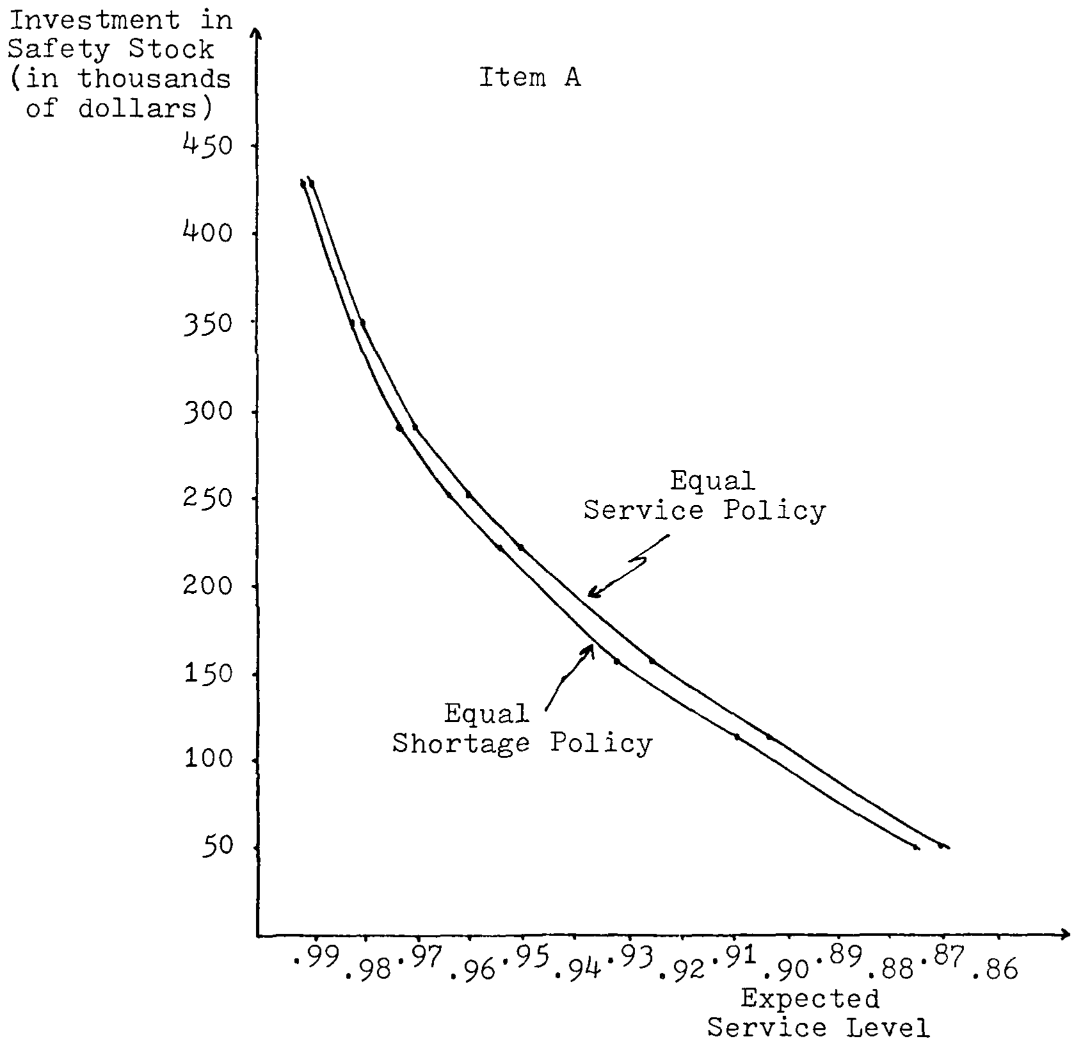
The order quantity, with a given safety factor, is deter-

mined by the standard deviation of leadtime demand. The larger the variance of demand, and the standard deviation of demand, the larger the order quantity. The standard deviations of leadtime demand for the items studied are relatively small when compared to the expected leadtime demands. Consequently, the derived order quantities are less than the expected leadtime demands (recall the safety factor computation for the .85 equal service level for item A1, p.41). With the level of uncertainty low, the order quantities must take into consideration the leadtime. The second case does not consider leadtime, consequently, with the order quantities smaller than the expected leadtime demands there is no clear method of computing the interval between orders. With the interval between orders not known, the standard deviation of demand for that period is not known. The problem becomes circular because without knowing the standard deviation, the order quantity cannot be computed. Conversely,without knowing the order quantity, the period over which to compute the standard deviation is not known. So, because of the uncertainty of demand being relatively small and the lack of consideration of leadtime the second case provides no clear solutions for the available data. The third case considers order costs and backorder costs which were ignored by the first and second cases. But because the order costs of the firm studied are not known and because of the same difficulties with the relatively

small variance of demand, the third case also cannot be applied to the study data.

E. Alternative Cost Curves

Comparison of the two methods of allocating inventory can be accomplished in a number of ways depending upon the particular aspects which the holder of the inventory is primarily concerned. The investment in safety stock, average inventory or cycle stock can all be contrasted at the levels of expected service for the different methods. The primary objective would be to accomplish the goals of the individual firm at the lowest possible cost. To compare the equal service and equal shortage policies for the items studied, curves depicting the relationship between the investment in safety stock and the expected level of service were constructed (see Figures III-4 & III-5). The same relationship that is shown also exists for contrasting expected service with average inventory, the level of safety stock, or the investment in average inventory. The results indicate that with the same amount of investment in safety stock the equal shortage policy will provide a slightly higher level of expected service. Equivalently, the equal shortage policy will provide the same service level as an equal service policy, but at a lower investment level. The differences will become negligible at the extremes of either high or low levels of safety stock investment. Either ap-



Investment in Safety Stock (in thousands of dollars)
Item A
450
400
350
300
250
200
150
100
50
Equal Service Policy
Equal Shortage Policy
.99
.98
.97
.96
.95
.94
.93
.92
.91
.90
.89
.88
.87
.86
Expected Service Level

Figure III-4

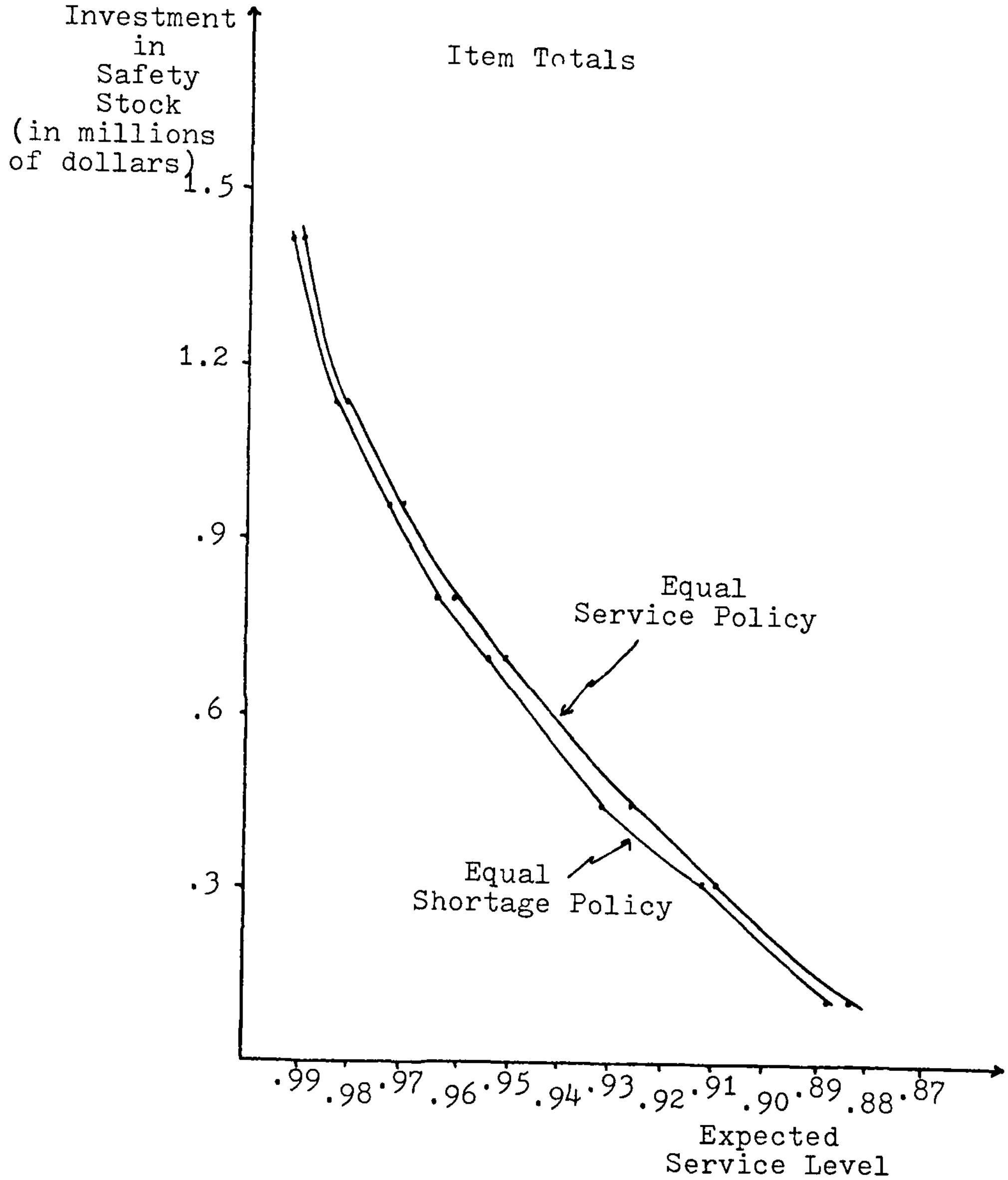
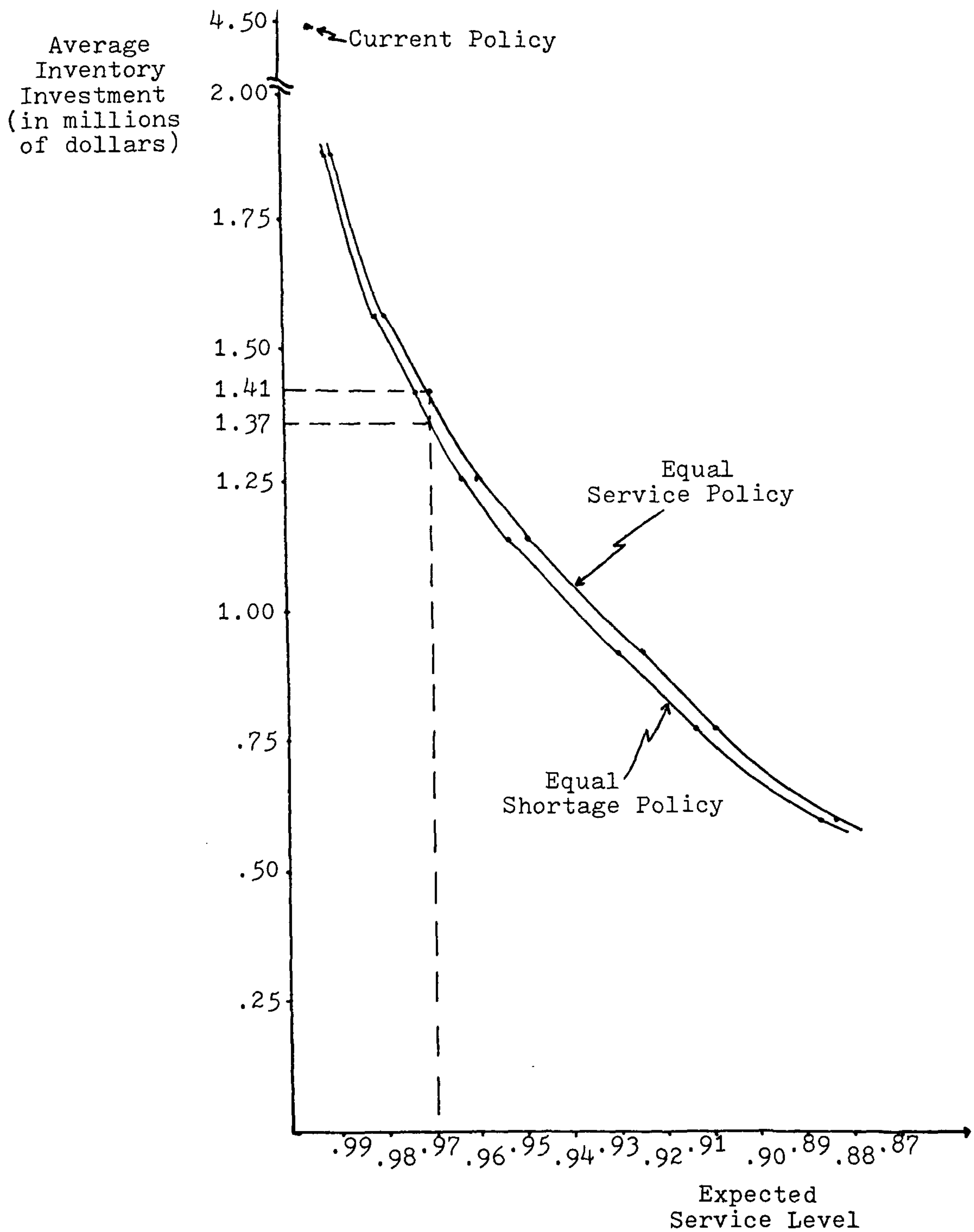

Investment in Safety Stock (in millions of dollars)
Item Totals
1.5
1.2
.9
.6
.3
Equal Service Policy
Equal Shortage Policy
.99 .98 .97 .96 .95 .94 .93 .92 .91 .90 .89 .88 .87
Expected Service Level

Figure III-5

proach would result in a substantial reduction in the current average inventory and, consequently, the investment required for inventory. The average inventory required for levels of service computed for the three policies (current, equal service, and equal shortage) can be compared. The levels of service provided by the three policies were not identical but were close enough so that comparisons of relative magnitude, while not exact, indicated differences in the resulting inventory investment. The respective levels of service and average inventories were:

Policy	Service Level	Average Inventory
Current	.993	4.47 million
Equal service	.99	1.89 million
Equal shortage	.9908	1.89 million

The actual costs of ordering and holding inventory could not be determined within the scope of this study. However, average inventory investment for various service levels using equal service and equal shortage policies can be readily compared in Figure III-6. For example, if the firm decided upon a .97 service level, the average inventory investment would be $1.41 million for an equal service policy and $1.37 million for an equal shortage policy. The trade-off between the investment in average inventory and the level of service becomes readily apparent for the two policies. This can be contrasted to the current average inventory investment of $4.47 million (at a $1 price). Although many factors may affect the size of an actual



Average Inventory Investment (in millions of dollars)
4.50
Current Policy
2.00
1.75
1.50
1.41
1.37
1.25
1.00
.75
.50
.25
Equal Service Policy
Equal Shortage Policy
.99 .98 .97 .96 .95 .94 .93 .92 .91 .90 .89 .88 .87
Expected Service Level

Figure III-6

reduction in average inventory investment, a reduction an order of magnitude less than this study indicated would still be substantial.

IV. Conclusions

The purpose of this thesis to examine objective methods by which an industrial firm could evaluate alternative inventory policies was achieved. It is evident that the firm which provided the data is providing significantly higher levels of customer service than was stated as its customer service level objective. By applying the techniques proposed in this thesis to its inventories the firm should be able to significantly reduce its investment in inventories, while decreasing its level of customer service only slightly, if at all. If the value of the inventory investment in the distribution warehouses totaled $50 million, 99 percent expected service, using an equal service policy,(99.08 percent expected service using an equal shortage policy) could be provided with an inventory investment reduced to $21.15 million. This would be a one time reduction in inventory investment of $28.85 million and a decrease in annual inventory carrying costs (at 18 percent) of $5.19 million. This, of course, is hypothetical since actual cost figures could not be determined for this study. It is realized that the firm may want to maintain a higher level of customer service than their competitors, but the level of customer service currently provided could still be maintained while achieving substantial reductions in inventories. Stockouts would occasionally occur, as they do now, but the costs of expediting occasional shipments

from alternative warehouses would not be nearly as substantial as the reduction in inventory costs. Ordering costs should be determined so that an optimal order quantity could be used to balance the inventory carrying costs. This would assist in ensuring that the lowest total inventory costs were achieved.

The difference between the equal service and equal shortage policies is significant between the extremes of high and low customer service. The equal shortage policy will provide the same expected customer service level as the equal service policy, but with a reduced investment in inventory. The specific data did not permit application of the second and third cases of the equal shortage policy. Further research into these cases is needed. Ordering costs, the relative magnitude of the demand variance, and the inclusion of significant leadtime should be prime concerns when considering data to analyze these cases.

The overall objectives of this thesis were achieved by applying an objective means of evaluating alternative inventory policies for an industrial firm. The procedures used here can be applied in similar circumstances.

APPENDIX A

Current Inventory List

Item	On-hand	In-transit	On-order	Current Inventory	Target
A1	319425		64000	383425	420000
A2	120549	24762		145311	140000
A3	19116			19116	20000
A4	102625			102625	100000
A5	64475			64475	70000
A6	65568			65568	68000
A7	54324		40000	94324	80000
A8	61417			61417	54000
A9	195126			195126	207000
A10	92545		40000	132545	140000
A11	59611		40000	99611	108000
A12	60184		44000	104184	110000
A13	40580	41050	40000	121630	105000
A14	61876			61876	60000
A15	308736	12795	40000	361531	390000
A16	23011			23011	34000
	1649168	78607	308000	2035775	2106000
B1	145494	16729	40000	202223	200000
B2	365713	121542	116000	603255	660000
B3	105317		40000	145317	180000
B4	57149		8000	65149	76000
B5	32429		8000	40429	40000
B6	76534		4000	80530	80000
B7	38869			38869	36000
B8	82249		28000	110249	100000
B9	55945			55945	66000
B10	46680	12568	32000	91248	80000
B11	88886			88886	80000
B12	198975			198975	200000

B13	69481		44000	113481	110000
B14	105021	1061		106082	110000
B15	64650	21689	8000	94339	100000
B16	147645			147645	140000
B17	13177			13177	15000
B18	146711		40000	186711	200000
	1840925	173589	368000	2382514	2473000
C1	405670		28000	433670	200000
C2	100438		84000	184438	200000
C3	46454			46454	52000
C4	64116			64116	84000
C5	127448		12000	139448	170000
C6	51151			51151	60000
C7	54423		16000	70423	72000
C8	49451		32000	81451	76000
C9	47784	14253		62037	75000
C10	166824	46274		213098	195000
C11	128449	49087	76000	253536	200000
C12	69111		32000	101111	110000
C13	128020		40000	168020	150000
C14	119386	37252		156638	150000
C15	145635		140000	285835	300000
C16	7875	7309		15184	16000
	1712235	154175	460000	2326410	2110000
Totals	5202328	406371	1136000	6744699	6689000

APPENDIX B

Item	Chi-Square Statistic	Type I Error Level Associated with Hypothesis*
A1	3.0	.01
A2	2.6	.01
A6	6.1	.03
B2	5.0	.01
B5	4.0	.01
B18	1.7	.01
C1	2.1	.01
C6	5.1	.01
C14	5.1	.01

*Hypothesis: Data is normally distributed.

APPENDIX C

Item Inventory Statistics

Item	Expected Leadtime Demand $\overline{X}L$	Standard Deviation of Leadtime Demand $\sigma_{\overline{X}L}$	Expected Weekly Demand D_w	Implied Safety Factor k $\frac{3Q}{\sigma_{\overline{X}L}}$
A1	89940	33000	29980	2.73
A2	42050	15970	14020	2.63
A3	4720	2280	1570	2.07
A4	41290	13150	13760	3.14
A5	24610	9070	8200	2.71
A6	28620	14300	9540	2.00
A7	18050	9860	6020	1.83
A8	15800	8130	5270	1.94
A9	87320	17150	29110	5.09
A10	56170	19190	18720	2.93
A11	16670	13260	5560	1.26
A12	41810	33350	13940	1.25
A13	40150	10360	13380	3.87
A14	15720	17570	5240	.89
A15	139160	44100	46390	3.16
A16	5490	6000	1830	.92
	667570	266740	222530	
B1	62440	29110	20810	2.14
B2	546630	98290	182210	5.56
B3	52730	13850	17580	3.80
B4	31090	11260	10360	2.76
B5	25220	20000	8410	1.26
B6	33660	12080	11220	2.79
B7	17200	8550	5730	2.01
B8	25440	10140	8480	2.51
B9	24710	11060	8240	2.24
B10	42710	8130	14240	5.25

B11	36620	6840	12210	5.35
B12	68200	18240	22730	3.74
B13	36890	15720	12300	2.35
B14	32780	19340	10930	1.70
B15	37200	12970	12400	2.87
B16	25040	15110	8350	1.66
B17	3750	2550	1250	1.47
B18	77500	25100	25830	3.09
	1179810	338340	393280	
C1	72350	26110	24120	2.77
C2	71350	23530	23780	3.03
C3	32490	7100	10830	4.58
C4	41770	27810	13920	1.50
C5	80860	47910	26950	1.69
C6	18780	4650	6260	4.04
C7	20900	10260	6970	2.04
C8	27180	8770	9060	3.10
C9	64620	29050	21540	2.22
C10	71700	13120	23900	5.46
C11	61520	23030	20510	2.67
C12	38840	12730	12950	3.05
C13	52110	29270	17370	1.78
C14	48990	24300	16330	2.02
C15	116310	27480	38770	4.23
C16	5340	7940	1780	.67
	825110	323060	274680	
Totals	2672490	928140	890490	

APPENDIX D

Current Item Safety Factors and Service Levels

Item	Safety Stock SS	Safety Factor k	Partial Expectation E(k)	Expected Shortage $E(k)\sigma_{\bar{X}L}$	Service Level P
A1	330060	10.0	--	--	1.000
A2	97940	6.13	--	--	1.000
A3	15290	6.71	--	--	1.000
A4	58720	4.47	--	--	1.000
A5	45400	5.01	--	--	1.000
A6	39380	2.75	.0009	13	.9995
A7	61940	6.28	--	--	1.000
A8	38190	4.70	--	--	1.000
A9	119670	6.98	--	--	1.000
A10	83840	4.37	--	--	1.000
A11	91320	6.89	--	--	1.000
A12	68180	2.04	.0076	253	.994
A13	64860	6.26	--	--	1.000
A14	44280	2.52	.0019	33	.998
A15	257580	5.84	--	--	1.000
A16	28510	4.75	--	--	1.000
	1445160			299	.9996
B1	137570	4.73	--	--	1.000
B2	113370	1.15	.062	6094	.989
B3	127260	9.19	--	--	1.000
B4	44920	3.99	--	--	1.000
B5	14770	.74	.133	2660	.895
B6	46340	3.84	--	--	1.000
B7	18810	2.20	.005	43	.9975
B8	74560	7.35	--	--	1.000
B9	41280	3.73	--	--	1.000
B10	37280	4.59	--	--	1.000
B11	43370	6.34	--	--	1.000

B12	131810	7.23	--	--	1.000
B13	73100	4.65	--	--	1.000
B14	77210	3.99	--	--	1.000
B15	62800	4.84	--	--	1.000
B16	114950	7.61	--	--	1.000
B17	11250	4.41	--	--	1.000
B18	122510	4.88	--	--	1.000
	1293160			8797	.9925
C1	1384940	53.04	--	--	1.000
C2	128660	5.47	--	--	1.000
C3	19510	2.75	.0009	6	.9998
C4	42240	1.52	.028	779	.981
C5	89150	1.86	.012	575	.993
C6	41220	8.86	--	--	1.000
C7	51090	4.98	--	--	1.000
C8	48820	5.57	--	--	1.000
C9	10380	.36	.245	7117	.89
C10	120300	9.17	--	--	1.000
C11	138470	6.01	--	--	1.000
C12	71150	5.60	--	--	1.000
C13	97890	3.34	.0001	3	.99994
C14	101010	4.16	--	--	1.000
C15	183690	6.68	--	--	1.000
C16	10660	1.34	.042	333	.937
	1285060			8813	.989
Totals	4023380			17909	.9933

APPENDIX E

Equal Service Safety Stocks and Targets

.99 Equal Service Level

Item	Partial Expectation E(k)	Safety Factor k	Safety Stock $k\sigma_{\overline{XL}}$	Target I_t
A1	.0273	1.53	50490	140430
A2	.0263	1.55	24750	66800
A3	.0207	1.65	3760	8480
A4	.0314	1.47	19330	60620
A5	.0271	1.53	13880	38490
A6	.0200	1.66	23740	52360
A7	.0183	1.70	16760	34810
A8	.0194	1.68	13660	29460
A9	.0509	1.25	21440	108760
A10	.0293	1.50	28790	84960
A11	.0126	1.85	24530	41200
A12	.0125	1.85	61700	103510
A13	.0387	1.37	14190	54340
A14	.0089	1.98	34790	50510
A15	.0316	1.47	64830	203990
A16	.0092	1.97	11820	17310
			428460	1096030
B1	.0214	1.63	47450	109890
B2	.0556	1.20	117950	664580
B3	.0380	1.38	19110	71840
B4	.0276	1.53	17230	48320
B5	.0126	1.85	37000	62220
B6	.0279	1.52	18360	52020
B7	.0201	1.66	14190	31390
B8	.0251	1.57	15920	41360
B9	.0224	1.62	17920	42630
B10	.0525	1.23	10000	52710
B11	.0535	1.22	8340	44960
B12	.0374	1.39	25350	93550
B13	.0235	1.60	25150	62040
B14	.0170	1.73	33460	66240
B15	.0287	1.51	19580	56780
B16	.0166	1.74	26920	51330
B17	.0147	1.79	4560	8310
B18	.0309	1.48	37150	114650
			495010	1674820
C1	.0277	1.52	39690	112040
C2	.0303	1.48	34820	106170
C3	.0458	1.30	9230	41270

C4	.0150	1.78	49500	91270
C5	.0169	1.73	82880	163740
C6	.0404	1.36	6320	25100
C7	.0204	1.65	16930	37830
C8	.0310	1.47	12890	40070
C9	.0222	1.62	47060	111680
C10	.0546	1.21	15880	87580
C11	.0267	1.54	35470	96990
C12	.0305	1.48	18840	57680
C13	.0178	1.71	50050	102160
C14	.0202	1.66	40340	89330
C15	.0423	1.33	36550	152860
C16	.0067	2.09	16590	21930
			513040	1338150
	.99 Totals		1436510	4109000

.98 Equal Service Level

A1	.0546	1.21	39930	129870
A2	.0526	1.23	19640	61690
A3	.0414	1.34	3060	7780
A4	.0628	1.14	14990	56280
A5	.0542	1.22	11070	35680
A6	.0400	1.36	19450	48070
A7	.0366	1.40	13800	31850
A8	.0388	1.37	11140	26940
A9	.1018	.89	15260	102580
A10	.0586	1.18	22640	78810
A11	.0252	1.56	20690	37360
A12	.0250	1.57	52360	94170
A13	.0774	1.04	10770	50920
A14	.0178	1.71	30040	45760
A15	.0632	1.14	50270	189430
A16	.0184	1.70	10200	15690
			345310	1012880
B1	.0428	1.33	38720	101160
B2	.1112	.85	83550	630180
B3	.0760	1.05	14540	67270
B4	.0552	1.21	13620	44710
B5	.0252	1.57	31400	56620
B6	.0558	1.20	14500	48160
B7	.0402	1.36	11630	28830
B8	.0502	1.25	12680	38120
B9	.0448	1.31	14490	39200
B10	.1505	.88	7150	49860
B11	.1070	.87	5950	42570
B12	.0748	1.06	19330	87530
B13	.0470	1.28	20120	57010
B14	.0340	1.43	27660	60440
B15	.0574	1.19	15430	52630

B16	.0332	1.44	21760	46800
B17	.0294	1.50	3830	7580
B18	.0618	1.15	28870	106370
			385230	1565040
C1	.0554	1.21	31590	103940
C2	.0606	1.16	27290	98640
C3	.0916	.95	6750	39240
C4	.0300	1.49	41440	83210
C5	.0338	1.44	68990	149850
C6	.0808	1.02	4740	23520
C7	.0408	1.35	13850	34750
C8	.0620	1.15	10090	37270
C9	.0444	1.31	38060	102680
C10	.1092	.85	11150	82850
C11	.0534	1.22	28100	89620
C12	.0610	1.16	14770	53610
C13	.0356	1.41	41270	93380
C14	.0404	1.36	33050	82040
C15	.0846	.99	27210	145320
C16	.0134	1.82	14450	19790
			412800	1237910
	.98 Totals		1143340	3815830

.97 Equal Service Level

A1	.0819	1.01	33330	123270
A2	.0789	1.03	16450	58500
A3	.0621	1.15	2620	7340
A4	.0942	.93	12230	53520
A5	.0813	1.01	9160	33770
A6	.0600	1.17	16730	45350
A7	.0549	1.21	11930	29980
A8	.0582	1.18	9590	25390
A9	.1527	.66	11320	98640
A10	.0879	.97	18610	74780
A11	.0378	1.39	18430	35100
A12	.0375	1.39	46360	88170
A13	.1161	.82	8500	48650
A14	.0267	1.54	27060	42780
A15	.0948	.93	41010	180170
A16	.0276	1.52	9120	14610
			292450	960020
B1	.0642	1.13	32890	95330
B2	.1668	.61	59960	606590
B3	.1140	.83	11500	64230
B4	.0828	1.00	11260	42350
B5	.0378	1.39	27800	53020

B6	.0837	1.00	12080	45740
B7	.0603	1.16	9920	27120
B8	.0753	1.05	10650	36090
B9	.0672	1.11	12280	36990
B10	.1575	.64	5200	47910
B11	.1605	.63	4310	40930
B12	.1122	.84	15320	83520
B13	.0705	1.09	17130	54020
B14	.0510	1.25	24180	56960
B15	.0861	.98	12710	49910
B16	.0498	1.26	19040	44080
B17	.0441	1.31	3340	7090
B18	.0927	.94	23590	101090
			313160	1492970
C1	.0831	1.00	26110	98460
C2	.0909	.95	22350	93700
C3	.1374	.72	5110	37600
C4	.0450	1.31	36430	78200
C5	.0507	1.25	59890	140750
C6	.1212	.79	3670	22450
C7	.0612	1.16	11900	32800
C8	.0930	.94	8240	35420
C9	.0666	1.11	32250	96870
C10	.1638	.62	8130	79830
C11	.0801	1.02	23490	85010
C12	.0915	.95	12090	50930
C13	.0534	1.22	35710	87820
C14	.0606	1.16	28190	77180
C15	.1269	.77	21160	137470
C16	.0201	1.66	13180	18520
			347900	1173010
	.97 Totals		953510	3626000

.96 Equal Service Level

A1	.1092	.85	28050	117990
A2	.1052	.87	13890	55940
A3	.0828	1.00	2280	7000
A4	.1256	.77	10130	51420
A5	.1084	.86	7800	32410
A6	.0800	1.02	14590	43210
A7	.0732	1.07	10550	28600
A8	.0776	1.04	8460	24260
A9	.2036	.48	8230	95550
A10	.1172	.81	15540	71710
A11	.0504	1.25	16580	33250
A12	.0500	1.26	42020	83830
A13	.1548	.65	6730	46880

A14	.0356	1.41	24770	40490
A15	.1264	.77	33960	173120
A16	.0368	1.40	8400	13890
			251980	919550
B1	.0856	.99	28820	91260
B2	.2224	.42	41280	587910
B3	.1520	.66	9140	61870
B4	.1104	.85	9570	40660
B5	.0504	1.25	25000	50220
B6	.1116	.84	10150	43810
B7	.0804	1.02	8720	25920
B8	.1004	.90	9130	34570
B9	.0896	.96	10620	35330
B10	.2100	.46	3740	46450
B11	.2140	.45	3080	39700
B12	.1496	.67	12220	80420
B13	.0940	.94	14780	51670
B14	.0680	1.10	21270	54050
B15	.1148	.83	10770	47970
B16	.0664	1.12	16920	41960
B17	.0588	1.18	3010	6760
B18	.1236	.78	19580	97080
			257800	1437610
C1	.1108	.85	22190	94540
C2	.1212	.79	18590	89940
C3	.1832	.55	3910	36400
C4	.0600	1.17	32540	74310
C5	.0676	1.11	53180	134040
C6	.1616	.63	2930	21710
C7	.0816	1.01	10360	31260
C8	.1240	.78	6840	34020
C9	.0888	.97	28180	92800
C10	.2184	.44	5770	77470
C11	.1068	.87	20040	81560
C12	.1220	.79	10060	48900
C13	.0712	1.08	31610	83720
C14	.0808	1.02	24790	73780
C15	.1692	.60	16490	132800
C16	.0268	1.55	12310	17650
			299790	1124900
	.96 Totals		809570	3482060

.95 Equal Service Level

A1	.1365	.73	24090	114030
A2	.1315	.75	11980	54030
A3	.1035	.88	2010	6730

A4	.1570	.64	8420	49710
A5	.1355	.73	6620	31230
A6	.1000	.90	12870	41490
A7	.0915	.95	9370	27420
A8	.0970	.92	7480	23280
A9	.2545	.33	5660	92980
A10	.1465	.68	13050	69220
A11	.0630	1.14	15120	31790
A12	.0625	1.15	38350	80160
A13	.1935	.51	5280	45430
A14	.0445	1.31	23020	38740
A15	.1580	.64	28220	167380
A16	.0460	1.29	7740	13230
			219280	886850
B1	.1070	.86	25030	87470
B2	.2780	.27	26540	573170
B3	.1900	.53	7340	60070
B4	.1380	.72	8110	39200
B5	.0630	1.14	22800	48020
B6	.1395	.71	8580	42240
B7	.1005	.88	7520	24720
B8	.1255	.78	7910	33350
B9	.1120	.84	9290	34000
B10	.2625	.31	2520	45230
B11	.2675	.30	2050	38670
B12	.1870	.53	9670	77870
B13	.1174	.81	12730	49620
B14	.0850	.99	19150	51930
B15	.1435	.70	9080	46280
B16	.0830	1.00	15110	40150
B17	.0735	1.06	2700	6450
B18	.1545	.65	16320	93820
			212450	1392260
C1	.1385	.72	18800	91150
C2	.1515	.67	15770	87120
C3	.2290	.40	2840	35330
C4	.0750	1.05	29200	70970
C5	.0845	.99	47430	128290
C6	.2020	.49	2280	21060
C7	.1020	.89	9130	30030
C8	.1550	.65	5700	32880
C9	.1110	.85	24690	89310
C10	.2730	.28	3670	75370
C11	.1335	.74	17040	78560
C12	.1525	.66	8400	47240
C13	.0890	.96	28100	80210
C14	.1010	.90	21870	70860
C15	.2112	.46	12640	128950
C16	.0335	1.44	11430	16770
			258990	1084100

.95 Totals			690720	3363210

.925 Equal Service Level

A1	.2047	.48	15840	105780
A2	.1972	.50	7990	50040
A3	.1552	.65	1480	6200
A4	.2355	.39	5130	46420
A5	.2032	.48	4350	28960
A6	.1500	.67	9580	38200
A7	.1372	.72	7100	25150
A8	.1455	.69	5610	21410
A9	.3817	.03	510	87830
A10	.2197	.43	8250	64420
A11	.0945	.93	12330	29000
A12	.0937	.94	31350	73160
A13	.2902	.24	2490	42640
A14	.0667	1.12	19680	35400
A15	.2370	.38	16760	155920
A16	.0690	1.10	6600	12090
			155050	822620
B1	.1605	.63	18340	80780
B2	.4170	--	---	546630
B3	.2850	.25	3460	56190
B4	.2070	.47	5290	36380
B5	.0945	.93	18600	43820
B6	.2092	.46	5560	39220
B7	.1507	.67	5730	22930
B8	.1882	.53	5370	30810
B9	.1680	.60	6640	31350
B10	.3937	.01	80	42790
B11	.4012	--	---	36620
B12	.2805	.26	4740	72940
B13	.1762	.57	8960	45850
B14	.1275	.77	14890	47670
B15	.2152	.45	5840	43040
B16	.1245	.78	11790	36830
B17	.1102	.85	2170	5920
B18	.2317	.40	10040	87540
			127500	1307310
C1	.2077	.47	12270	84620
C2	.2272	.41	9650	81000
C3	.3435	.12	850	33340
C4	.1125	.84	23360	65130
C5	.1267	.77	36890	117750
C6	.3030	.21	980	19760
C7	.1530	.66	6770	27670

C8	.2325	.39	3420	30600
C9	.1665	.61	17720	82340
C10	.4095	--	---	71700
C11	.2002	.49	11280	72800
C12	.2287	.42	5350	44190
C13	.1335	.74	21660	73770
C14	.1515	.67	16280	65270
C15	.3168	.18	4950	121260
C16	.0502	1.25	9930	15270
			181360	1006470
	.925 Totals		463910	3136400

.90 Equal Service Level

A1	.2730	.28	9240	99180
A2	.2630	.31	4950	47000
A3	.2070	.47	1070	5790
A4	.3140	.18	2370	43660
A5	.2710	.29	2630	27240
A6	.2000	.49	7010	35630
A7	.1830	.55	5420	23470
A8	.1940	.51	4150	19950
A9	.5090	--	---	87320
A10	.2930	.23	4410	60580
A11	.1260	.77	10210	26880
A12	.1250	.78	26010	67820
A13	.3870	.02	210	40360
A14	.0890	.96	16870	32590
A15	.3160	.18	7940	147100
A16	.0920	.95	5700	11190
			108190	775760
B1	.2140	.45	13100	75540
B2	.5560	--	---	546630
B3	.3800	.04	550	53280
B4	.2760	.28	3150	34240
B5	.1260	.77	15400	40620
B6	.2790	.27	3260	36920
B7	.2010	.49	4190	21390
B8	.2510	.34	3450	28890
B9	.2240	.42	4650	29360
B10	.5250	--	---	42710
B11	.5350	--	---	36620
B12	.3740	.05	910	69110
B13	.2350	.39	6130	43020
B14	.1700	.59	11410	44190
B15	.2870	.25	3240	40440
B16	.1660	.61	9220	34260

B17	.1470	.68	1730	5480
B18	.3090	.20	5020	82520
			85410	1265220
C1	.2770	.27	7050	79400
C2	.3030	.21	4940	76290
C3	.4580	--	---	32490
C4	.1500	.67	18630	60400
C5	.1690	.60	28750	109610
C6	.4040	--	---	18780
C7	.2040	.48	4920	25820
C8	.3100	.19	1670	28850
C9	.2220	.42	12200	76820
C10	.5460	--	---	71700
C11	.2670	.30	6910	68430
C12	.3050	.20	2550	41390
C13	.1780	.57	16680	68790
C14	.2020	.49	11910	60900
C15	.4230	--	---	116310
C16	.0670	1.11	8810	14150
			125020	950130
	.90 Totals		318620	2991110

.85 Equal Service Level

A1	.4095	--	---	89940
A2	.3945	.01	160	42210
A3	.3105	.19	430	5150
A4	.4710	--	---	41290
A5	.4065	--	---	24610
A6	.3000	.22	3150	31770
A7	.2745	.28	2760	20810
A8	.2910	.24	1950	17750
A9	.7635	--	---	87320
A10	.4395	--	---	56170
A11	.1890	.53	7030	23700
A12	.1875	.53	17680	59490
A13	.5805	--	---	40150
A14	.1335	.74	13000	28720
A15	.4740	--	---	139160
A16	.1380	.72	4320	9810
			50480	718050
B1	.3210	.17	4950	67390
B2	.8340	--	---	546630
B3	.5700	--	---	52730
B4	.4140	--	---	31090
B5	.1890	.53	10600	35820
B6	.4184	--	---	33660

B7	.3014	.21	1800	19000
B8	.3764	.05	510	25950
B9	.3360	.13	1440	26150
B10	.7874	--	---	42710
B11	.8024	--	---	36620
B12	.5610	--	---	68200
B13	.3524	.10	1570	38460
B14	.2550	.33	6380	39160
B15	.4304	--	---	37200
B16	.2490	.35	5290	30330
B17	.2204	.43	1100	4850
B18	.4634	--	---	77500
			33640	1213450
C1	.4154	--	---	72350
C2	.4544	--	---	71350
C3	.6870	--	---	32490
C4	.2250	.42	11680	53450
C5	.2534	.34	16290	97150
C6	.6060	--	---	18780
C7	.3060	.20	2050	22950
C8	.4650	--	---	27180
C9	.3330	.14	4070	68690
C10	.8190	--	---	71700
C11	.4004	--	---	61520
C12	.4574	--	---	38840
C13	.2670	.30	8780	60890
C14	.3030	.21	5100	54090
C15	.6336	--	---	116310
C16	.1004	.90	7150	12490
			55120	880230
	.85 Totals		139240	2811730

APPENDIX F

Average Inventory for the Current and for each Equal Service Level

Item	Current	.99	.98	.97	.96	.95	.925	.90	.85
A1	345050	65480	54920	48320	43040	39080	30830	24230	14990
A2	104950	31750	26640	23450	20890	18980	14990	11950	7160
A3	16075	4555	3855	3415	3075	2805	2275	1865	1225
A4	65600	26220	21880	19120	17020	14770	12020	9260	6890
A5	49500	17990	15180	13270	11910	10730	8460	6740	4110
A6	44150	28510	24220	21500	19360	17640	14350	11780	7920
A7	64950	19760	16800	14930	13550	12370	10100	8420	5760
A8	40825	16285	13765	12215	11085	10105	8235	6775	4575
A9	134225	35985	29805	25865	22775	20205	15055	14545	14545
A10	93200	38160	32010	27980	24910	22420	17620	13780	9370
A11	94100	27300	23460	21200	19350	17890	15100	12980	9800
A12	75150	68660	59320	53320	48980	45310	38310	32970	24640
A13	71550	20890	17470	15200	13430	11980	10190	6910	6700
A14	46900	37410	32660	29680	27390	25640	22300	19490	15620
A15	280775	94765	80205	70945	63895	58155	46695	37875	29935
A16	29425	12735	11115	10035	9315	8655	7515	6615	5235
	1556425	546455	463305	410445	369975	337275	273045	226045	168475
Percent of Current	100.0	35.1	29.8	26.4	23.8	21.7	17.5	14.5	10.8
B1	147975	57865	49135	43305	39235	35445	28755	23515	15365
B2	204475	209055	174655	151065	132385	117645	91105	91105	91105
B3	136050	27890	23320	20280	17920	16120	12240	9330	8780
B4	50100	22420	18810	16450	14760	13300	10480	8340	5190
B5	18975	41195	35595	31995	29195	26995	22795	19595	14795
B6	51950	23970	20110	17690	15130	14190	11170	8870	5610
B7	21675	17065	14505	12795	11595	10395	8605	7065	4675
B8	78800	20160	16920	14890	13370	12150	9610	7690	4750
B9	45400	22030	18600	16390	14730	13400	10750	8760	5550
B10	44400	17110	14260	12310	10850	9630	7190	7110	7110
B11	49475	14435	12045	10405	9175	8145	6095	6095	6095
B12	143175	36725	30705	26695	23595	21045	16115	12285	11375
B13	79250	31290	26260	23270	20920	18870	15100	12270	7710

B14	82675	38915	33115	29635	26725	24605	20345	16865	11835
B15	69000	25780	21630	18910	16970	15280	12040	9440	6200
B16	1191[illegible]5	30455	25925	23205	21085	19275	15955	13385	9455
B17	11875	5185	4455	3965	3635	3325	2795	2355	1725
B18	135425	50075	41795	36515	32505	29245	22965	17945	12925
	1489800	691620	581840	509770	454410	409060	324110	282020	230250
Percent of Current	100.0	46.4	39.1	34.2	30.5	27.5	21.8	18.9	15.5
C1	139700	51740	43640	38160	34240	30850	24320	19100	12050
C2	140550	46720	39190	34250	30490	27670	21550	16840	11900
C3	24925	14645	12165	10525	9325	8255	6265	5415	5415
C4	49200	56470	48410	43400	39510	36170	30330	25600	18650
C5	102625	96365	82475	73375	66665	60915	50375	42235	29775
C6	44350	9450	7870	6800	6060	5410	4110	3130	3130
C7	54575	20405	17325	15375	13835	12605	10245	8395	5525
C8	53350	17420	14620	12770	11370	10230	7950	6200	4530
C9	21150	57830	48830	43020	38950	35460	28490	22970	14840
C10	132250	27830	23100	20080	17720	15620	11950	11950	11950
C11	148725	45715	38345	33735	30285	27285	21525	17155	10245
C12	77625	25305	21235	18555	16525	14865	11815	9015	6465
C13	106575	58735	49955	44395	40295	36785	30345	25365	17465
C14	109175	49505	41215	36355	32955	30035	24445	20075	13265
C15	203075	55935	46595	40545	35875	32025	24335	19385	19385
C16	11550	17480	15340	14070	13200	12320	10820	9700	8040
	1422400	650550	550310	485410	437300	396500	318870	262530	192630
Percent of Current	100.0	45.7	38.7	34.1	30.7	27.9	22.4	18.5	13.5
Grand Totals	4468625	1888625	1595455	1405625	1261685	1142835	916025	770595	591355
Percent of Current	100.0	42.3	35.7	31.5	28.2	25.6	20.5	17.2	13.2

APPENDIX G

Equal Shortage Safety Factor Computations

Equal Service Level & Item	ISS/$\Sigma\sigma_{\overline{X}Lj}$	=	k	E(k)
.99/A	428460/266740		1.606	.0229
.98/A	345310/266740		1.295	.0470
.97/A	292450/266740		1.096	.0692
.96/A	251980/266740		.945	.0924
.95/A	219280/266740		.822	.1156
.925/A	155050/266740		.581	.1739
.90/A	108190/266740		.406	.2284
.85/A	50480/266740		.189	.3115
.99/B	495010/338340		1.463	.0319
.98/B	385230/338340		1.139	.0635
.97/B	313160/338340		.926	.0957
.96/B	257800/338340		.762	.1285
.95/B	212450/338340		.628	.1610
.925/B	127500/338340		.377	.2385
.90/B	85410/338340		.252	.2855
.85/B	33640/338340		.100	.3500
.99/C	513040/323060		1.588	.0239
.98/C	412800/323060		1.278	.0477
.97/C	347900/323060		1.077	.0718
.96/C	299790/323060		.928	.0954
.95/C	258990/323060		.802	.1198
.925/C	181360/323060		.561	.1796
.90/C	125020/323060		.387	.2350
.85/C	55120/323060		.171	.3193

APPENDIX H

Equal Shortage Safety Stocks and Targets

Item	k = 1.606 Safety Stock	Target	k = 1.295 Safety Stock	Target	k = 1.096 Safety Stock	Target
A1	53000	142940	42740	132680	36170	126110
A2	25650	67700	20680	62730	17500	59550
A3	3660	8380	2950	7670	2500	7220
A4	21120	62410	17030	58320	14410	55700
A5	14570	39180	11750	36360	9940	34550
A6	22970	51590	18520	47140	15670	44290
A7	15840	33890	12770	30820	10810	28660
A8	13060	28860	10530	26330	8910	24710
A9	27540	114860	22210	109530	18800	106120
A10	30820	86990	24850	81020	21030	77200
A11	21300	37970	17170	33840	14530	31200
A12	53560	95370	43190	85000	36550	78360
A13	16640	56790	13420	53570	11360	51510
A14	28220	43940	22750	38470	19250	34970
A15	70820	209980	57110	196270	48330	187490
A16	9640	15130	7770	13260	6580	12070
	428410	1095980	345440	1013010	292430	959910

Item	k = 1.463 Safety Stock	Target	k = 1.139 Safety Stock	Target	k = .926 Safety Stock	Target
B1	43760	106200	34070	96510	27700	90140
B2	143800	690430	111950	658580	91020	637650
B3	20260	72990	15780	68510	12830	65560
B4	16470	47560	12830	43920	10430	41520
B5	29260	54480	22780	48000	18520	43740
B6	17670	51330	13760	47420	11190	44850
B7	12510	29710	9740	26940	7920	25120
B8	14830	40270	11550	36990	9390	34830
B9	16180	40890	12600	37310	10240	34950
B10	11890	54600	9260	51970	7530	50240
B11	10010	46630	7790	44410	6330	42950
B12	26690	94890	20780	88980	16890	85090
B13	23000	59890	17910	54800	14560	51450
B14	28290	61070	22030	54810	17910	50690
B15	18980	56180	14780	51980	12010	49210
B16	22110	47150	17210	42250	13990	39030
B17	3730	7480	2900	6650	2360	6110
B18	36720	114220	28590	106090	23240	100740
	496160	1675970	386310	1566120	314060	1493870

	k = 1.588		k = 1.278		k = 1.077	
C1	41460	113810	33370	105720	28120	100470
C2	37370	108720	30070	101420	25340	96690
C3	11270	43760	9070	41560	7650	40140
C4	44160	85930	35540	77310	29950	71720
C5	76080	156940	61230	142090	51600	132460
C6	7380	26160	5940	24720	5010	23790
C7	16290	37190	13110	34010	11050	31950
C8	13930	41110	11210	38390	9450	36630
C9	46130	110750	37130	101750	31290	95910
C10	20830	92530	16770	88470	14130	85830
C11	36570	98090	29430	90950	24800	86320
C12	20220	59060	16270	55110	13710	52550
C13	46480	98590	37410	89520	31520	83630
C14	38590	87580	31060	80050	26170	75160
C15	43640	159950	35120	151430	29600	145910
C16	12610	17950	10150	15490	8550	13890
	513010	1338120	412880	1237990	347940	1173050
	1437580	4110070	1144630	3817120	954430	3626830
Equivalent Equal Service Level	.99		.98		.97	

	k = .945		k = .822		k = .581	
A1	31190	121130	27130	117070	19180	109120
A2	15090	57140	13130	55180	9280	51330
A3	2160	6880	1880	6600	1330	6050
A4	12420	53710	10800	52090	7630	48920
A5	8570	33180	7450	32060	5270	29880
A6	15670	42130	11750	40370	8310	36930
A7	9320	27370	8110	26160	5730	23780
A8	7680	23480	6680	22480	4720	20520
A9	16210	103530	14100	101420	9970	97290
A10	18130	74300	15770	71940	11150	67320
A11	12530	29200	10900	27570	7700	24370
A12	31510	73320	27410	69220	19180	60990
A13	9790	49940	8520	48670	6020	46170
A14	16600	32320	14440	30160	10210	25930
A15	41670	180830	36250	175410	25620	164780
A16	5670	11160	4930	10420	3480	8970
	252050	919620	219250	886820	154780	822350

	k = .762		k = .628		k = .377	
B1	22790	85230	18780	81220	11280	73720
B2	74900	621530	61730	608360	37060	583690
B3	10550	63280	8700	61430	5220	57950
B4	8580	39670	7070	38160	4250	35340
B5	15240	40460	12560	37780	7540	32760

B6	9200	42860	7590	41250	4550	38210
B7	6520	23720	5370	22570	3220	20420
B8	7730	33170	6370	31810	3820	29260
B9	8430	33140	6950	31660	4170	28880
B10	6200	48910	5110	47820	3070	45780
B11	5210	41830	4300	40920	2580	39200
B12	13900	82100	11450	79650	6880	75080
B13	11980	48870	9870	46760	5930	42820
B14	14740	47520	12150	44930	7290	40070
B15	9880	47080	8150	45350	4890	42090
B16	11510	36550	9490	34530	5700	30740
B17	1940	5690	1600	5350	960	4710
B18	19130	96630	15760	93260	9460	86960
	258430	1438240	213000	1392810	127870	1307680
	k =	.928	k =	.802	k =	.561
C1	24230	96580	20940	93290	14650	87000
C2	21840	93190	18870	90220	13200	84550
C3	6590	39080	5690	38180	3980	36470
C4	25810	67580	22300	64070	15600	57370
C5	44460	125320	38420	119280	26880	107740
C6	4320	23100	3730	22510	2610	21390
C7	9520	30420	8230	29130	5760	26660
C8	8140	35320	7030	34120	4920	32100
C9	26960	91580	23300	87920	16300	80920
C10	12180	83880	10520	82220	7360	79060
C11	21370	82890	18470	79990	12920	74440
C12	11810	50650	10210	49050	7140	45980
C13	27160	79270	23470	75580	16420	68530
C14	22550	71540	19490	68480	13630	62620
C15	25500	141810	22040	138350	15420	131730
C16	7370	12710	6370	11710	4450	9790
	299810	1124920	259080	1084190	181240	1006350
	810290	3482780	691330	3363820	463890	3136380
Equivalent Equal Service Level	.96		.95		.925	

	k =	.406	k =	.189
A1	13400	103340	6240	96180
A2	6480	48530	3020	45070
A3	930	5650	430	5150
A4	5330	46620	2480	43770
A5	3680	28290	1710	26320
A6	5810	34430	2700	31320
A7	4000	22050	1860	19910
A8	3300	19100	1540	17340
A9	6970	94290	3240	90560

A10	7790	63960	3630	59800
A11	5380	22050	2500	19170
A12	13400	55210	6240	48050
A13	4210	44360	1960	42110
A14	7130	22850	3320	19040
A15	17900	157060	8330	147490
A16	2430	7920	1130	6620
	108140	775710	50330	717900
	k = .252		k = .100	
B1	7540	69980	2990	65430
B2	24770	571400	9830	556460
B3	3490	56220	1390	54120
B4	2840	33930	1130	32220
B5	5040	30260	2000	27220
B6	3040	36700	1210	34870
B7	2150	19350	860	18060
B8	2560	28000	1010	26450
B9	2790	27500	1110	25820
B10	2050	44760	810	43520
B11	1720	38340	680	37300
B12	4600	72800	1820	70020
B13	3960	40850	1570	38460
B14	4870	37650	1930	34710
B15	3270	40470	1300	38500
B16	3810	28850	1510	26550
B17	640	4390	260	4010
B18	6330	83830	2510	80010
	85470	1265280	33920	1213730
	k = .387		k = .171	
C1	10100	82450	4460	76810
C2	9110	80460	4020	75370
C3	2750	35240	1210	33700
C4	10760	52530	4760	46530
C5	18540	99400	8190	89050
C6	1800	20580	800	19580
C7	3970	24870	1750	22650
C8	3390	30570	1500	28680
C9	11240	75860	4970	69590
C10	5080	76780	2240	73940
C11	8910	70430	3940	65460
C12	4930	43770	2180	41020
C13	11330	63440	5010	57120
C14	9400	58390	4160	53150
C15	10630	126940	4700	121010
C16	3070	8410	1360	6700
	125010	950120	55250	880360
	318620	2991110	139500	2811990
Equivalent Equal Service Level	.90		.85	

APPENDIX I

Expected Backorders for Separate Service Levels of Equal Service and Equal Shortage Policies with the Same Level of Inventory

	Expected Backorders		Expected Backorders	
Item	Equal Service	Equal Shortage	Equal Service	Equal Shortage
A1	901	756	1802	1551
A2	420	366	840	751
A3	47	52	94	107
A4	413	301	826	618
A5	246	208	492	426
A6	286	327	572	672
A7	180	226	361	463
A8	158	186	315	382
A9	873	393	1746	806
A10	562	439	1125	902
A11	167	304	334	623
A12	417	764	834	1567
A13	401	237	802	487
A14	156	402	313	826
A15	1394	1010	2787	2073
A16	55	137	110	282
	6676	6108	13353	12536
Expected Service Level	.99	.991	.98	.9812
B1	623	929	1246	1848
B2	5465	3135	10930	6241
B3	526	442	1053	879
B4	311	359	622	715
B5	252	638	504	1270
B6	337	385	674	767
B7	172	273	344	543
B8	255	323	509	644
B9	248	353	495	702
B10	426	259	854	516
B11	366	218	732	434
B12	682	582	1364	1158
B13	369	501	739	998
B14	329	620	658	1228
B15	372	414	744	824
B16	251	482	502	959
B17	37	81	75	162
B18	776	801	1551	1594
	11797	10795	23596	21482
Expected Service Level	.99	.9908	.98	.9818

C1	723	624	1446	1245
C2	713	562	1426	1122
C3	325	170	650	339
C4	417	665	834	1327
C5	810	1145	1619	2285
C6	188	111	376	222
C7	209	245	419	489
C8	272	210	544	418
C9	645	694	1290	1387
C10	716	314	1433	626
C11	615	550	1230	1099
C12	388	304	777	607
C13	521	700	1042	1396
C14	491	581	982	1159
C15	1162	657	2325	1311
C16	53	190	106	379
	8248	7722	16499	15411
Expected Service Level	.99	.9906	.98	.9813
Totals	26721	24625	53448	49429
	.99	.9908	.98	.9815
A1	2703	2284	3604	3049
A2	1260	1105	1680	1476
A3	142	158	189	211
A4	1239	910	1652	1215
A5	737	628	983	838
A6	858	990	1144	1321
A7	541	682	722	911
A8	473	563	631	751
A9	2619	1187	3492	1585
A10	1687	1328	2249	1773
A11	501	918	668	1225
A12	1251	2308	1668	3082
A13	1203	717	1604	957
A14	469	1216	625	1623
A15	4181	3052	5574	4075
A16	166	415	221	554
	20030	18461	26706	24646
Expected Service Level	.97	.9724	.96	.9631
B1	1869	2786	2492	3741
B2	16395	9406	21860	12630
B3	1579	1325	2105	1780
B4	932	1078	1243	1447
B5	756	1914	1008	2570
B6	1011	1156	1348	1552

B7	516	818	687	1099
B8	764	970	1018	1303
B9	743	1058	991	1421
B10	1280	778	1707	1045
B11	1098	655	1464	879
B12	2047	1746	2729	2344
B13	1108	1504	1478	2020
B14	986	1851	1315	2485
B15	1117	1241	1489	1667
B16	752	1446	1003	1942
B17	112	244	150	328
B18	2327	2402	3102	3225
	35392	32378	47189	43478
Expected Service Level	.97	.9726	.96	.9631
C1	2170	1875	2893	2491
C2	2139	1689	2852	2245
C3	976	510	1301	677
C4	1251	1997	1669	2653
C5	2429	3440	3239	4571
C6	564	334	751	444
C7	628	737	837	979
C8	816	630	1087	837
C9	1935	2086	2580	2771
C10	2149	942	2865	1252
C11	1845	1654	2460	2197
C12	1165	914	1553	1214
C13	1563	2102	2084	2792
C14	1473	1745	1963	2318
C15	3487	1973	4650	2622
C16	160	570	213	757
	24750	23198	32997	30820
Totals	80172	74037	106892	98944
	.97	.9723	.96	.9630

A1	4505	3815	6755	5739
A2	2100	1846	3149	2777
A3	236	264	354	396
A4	2065	1520	3097	2287
A5	1229	1048	1843	1577
A6	1430	1653	2145	2487
A7	902	1140	1353	1715
A8	789	940	1183	1414
A9	4365	1983	6546	2982
A10	2811	2218	4216	3337
A11	835	1533	1253	2306

A12	2084	3855	3125	5800
A13	2005	1198	3006	1802
A14	782	2031	1172	3055
A15	6968	5098	10452	7669
A16	276	694	414	1043
	33382	30836	50063	46386
Expected Service Level	.95	.9538	.925	.9305
B1	3115	4687	4672	6943
B2	27325	15825	39208	23442
B3	2632	2230	3947	3303
B4	1554	1813	2331	2686
B5	1260	3220	1890	4770
B6	1685	1945	2527	2881
B7	859	1377	1288	2039
B8	1273	1633	1908	2419
B9	1238	1781	1868	2638
B10	2134	1309	3201	1939
B11	1830	1101	2728	1631
B12	3411	2937	5116	4350
B13	1847	2531	2770	3749
B14	1644	3114	2466	4613
B15	1861	2088	2791	3093
B16	1254	2433	1881	3604
B17	187	411	281	608
B18	3878	4041	5816	5986
	58987	54476	86689	80694
Expected Service Level	.95	.9538	.9265	.9316
C1	3616	3128	5423	4689
C2	3565	2819	5346	4226
C3	1626	851	2439	1275
C4	2086	3332	3129	4995
C5	4048	5740	6070	8605
C6	939	557	1409	835
C7	1047	1229	1570	1843
C8	1359	1051	2039	1575
C9	3225	3480	4837	5217
C10	3582	1572	5234	2356
C11	3075	2759	4611	4136
C12	1941	1525	2911	2286
C13	2605	3507	3908	5257
C14	2454	2911	3681	4364
C15	5804	3292	8706	4935
C16	266	951	399	1426
	41238	38704	61712	58020
Expected Service Level	.95	.9531	.9252	.9297

Totals	133607	124016	198464	185100
	.95	.9536	.9257	.9307
A1	9009	7537	13164	10280
A2	4200	3648	6300	4975
A3	472	521	708	710
A4	4129	3003	5246	4096
A5	2458	2072	3618	2825
A6	2860	3266	4290	4454
A7	1804	2252	2707	3071
A8	1577	1857	2366	2532
A9	6841	3917	6841	5342
A10	5623	4383	7655	5978
A11	1671	3029	2506	4130
A12	4169	7617	6253	10389
A13	4009	2366	4133	3227
A14	1564	4013	2346	5473
A15	13936	10072	17591	13737
A16	552	1370	828	1869
	64874	60923	86552	83088
Expected Service Level	.903	.909	.8704	.8756
B1	6230	8311	9344	10189
B2	39208	28062	39208	34402
B3	5263	3954	5525	4848
B4	3108	3215	4492	3941
B5	2520	5710	3780	7000
B6	3370	3449	4819	4228
B7	1719	2441	2577	2993
B8	2545	2895	3817	3549
B9	2477	3158	3716	3871
B10	3243	2321	3243	2846
B11	2728	1953	2728	2394
B12	6822	5208	7276	6384
B13	3694	4488	5540	5502
B14	3288	5522	4932	6769
B15	3722	3703	5174	4540
B16	2508	4314	3762	5289
B17	375	728	562	893
B18	7756	7166	10012	8785
	100576	96598	120507	118423
Expected Service Level	.9147	.9181	.8979	.8996
C1	7232	6136	10415	8337
C2	7130	5530	9386	7513
C3	2832	1669	2832	2267
C4	4172	6535	6257	8880
C5	8097	11259	12140	15298

C6	1855	1093	1855	1485
C7	2093	2411	3140	3276
C8	2719	2061	3498	2800
C9	6449	6827	9674	9276
C10	5234	3083	5234	4189
C11	6149	5412	9187	7353
C12	3883	2992	5078	4065
C13	5210	6878	7815	9346
C14	4909	5711	7363	7759
C15	10962	6458	10962	8774
C16	532	1866	797	2535
	79458	75921	105633	103153
Expected Service Level	.9037	.9080	.8720	.8750
Totals	244908	233442	312692	304664
	.9084	.9127	.8830	.8860

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VITA

Raymond G. Friedl was born in Lakewood, Ohio on March 25, 1943. He received a Bachelor of Science in Business Administration fron The University of Tennessee in June 1973. He has been in the United States Air Force for eighteen years and in the transportation field since 1973. From 1973 through 1977 he served as a Vehicle Management Officer in the United States and in the United Kingdom. He is presently a transportation analyst and chief of the traffic division of the Air Force Logistics Management Center at Gunter Air Force Station in Montgomery, Alabama.

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